A signalling model of school grades:

centralized versus decentralized examinations

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The aim of the paper

This paper examines the signalling value of individual skills of different examination systems, for the labor market, in relation to errors that may affect evaluations obtained by students.

Two type of errors are possible: 1) errors that influence student performance; 2) errors deriving from different performance evaluation standards.

The relevance of these errors depends on the type of examination system adopted. We compare on efficiency grounds decentralized and centralized evaluation systems in relation to these errors.

We investigate the relationship between the optimal class size and evaluation systems.

Related Literature

A number of authors emphasize the positive consequences that centralised examinations may produce on agents involved in the educational process. (Woessman, 2005; Bishop Woessman, 2005; Bishop 1997, 1999; Lavy, 2002, 2003; Glewwe, Elias, and Kremer 2003; Jürges, Richter, and Schneider 2004).

In spite of the favour encountered by centralized exams among economists they are strongly criticized among teachers and pedagogical specialists, who question their efficacy, since they undermine educational freedom and the pedagogical discretion that is supposed to be necessary to deal with heterogeneity among students.

An important issue, which we analyse in this paper, is represented by the effect produced by different evaluation systems on measurement errors affecting students' evaluations at exams and the use of these evaluations as a signal of effective skills.

Organization

- We present a simple model showing how the signalling value of school grades on the labor market is influenced by the accuracy of evaluation systems.

- We analyse the effect that more precise evaluation systems produce on student effort and welfare.

- We compare advantages and disadvantages of centralized and decentralized evaluation systems.

- We discuss the relationship between class size and evaluation systems.

- Conclusions

The model

We assume that individuals are risk-neutral and live for two periods: in the first period they go to school, sustaining the cost of effort, and in the second period they enter the labor market, obtaining a wage *W*.

No discounting. Individuals are identical in every respect except their ability that is distributed according to a probability density function with mean $\bar{a}$ and variance σ_a^2.

Students attend school and attain an educational qualification with an evaluation of their skills made by schools.

[1] $$v_i = e_i + a_i + \varepsilon_i + \eta_i$$

$\mathrm{var}(\varepsilon) = \sigma_\varepsilon^2$**,** $\mathrm{var}(\eta) = \sigma_\eta^2$**,** $Cov(e,\eta) = Cov(e,\varepsilon) = Cov(a,\eta) = Cov(a,\varepsilon) = Cov(e,\eta) = Cov(\varepsilon,\eta) = 0$

$Var(v) = \sigma_a{}^2 + \sigma_\varepsilon{}^2 + \sigma_\eta{}^2 . E(v) = \bar{e}^* + \bar{a}$

Labour market

We assume that the output q_i produced by an individual in the labour market is related to his skills deriving from his innate ability and from the effort provided during the period he was attending school. Therefore, we suppose that skills are equal to $s_i = e_i + a_i$. Output is then related to skills according to the following production function:

[2] $$q_i = \pi s_i$$

where π is a productivity parameter.

Firms are not able to observe neither individuals' abilities neither the effort they provided in the educational process, but only observe the evaluation v_i obtained by each student. Firms pay a wage related to individual expected productivity.

Firms seek to infer the effective skills of workers, on the basis of the evaluation v_i.

They solve a typical signal extraction problem and therefore form an expectation of workers skills $E(s_i \mid v_i) = \beta_0 + \beta_1 v_i$, where the parameters β_0 and β_1 can be estimated using the standard OLS formulae.

It follows that the two parameters β_0 and β_1 are given by the following expressions:

[3] $$\beta_1 = \frac{Cov(s,v)}{Var(v)} = \frac{Cov(e+a, e+a+\varepsilon+\eta)}{Var(e+a+\varepsilon+\eta)} \qquad \beta_0 = E(s) - \beta_1 E(v)$$

Given our assumptions on variance and covariance of variables, it is possible to show that $Cov(s,v) = Var(a) = \sigma_a{}^2$. Therefore:

[4] $$\beta_1 = \frac{\sigma_a{}^2}{\sigma_a{}^2 + \sigma_\varepsilon{}^2 + \sigma_\eta{}^2} \qquad \beta_0 = \left(\bar{e}^* + \bar{a}\right)(1 - \beta_1)$$

the wage paid by employers is: $W = \pi(\beta_0 + \beta_1 v_i)$

Student behaviour

The lifetime individual utility function takes the following simple form:

[7] $$E(U_i) = E(W_i) - \frac{\gamma e_i^{\,2}}{2}$$

Since $W = \pi E(e + a \mid v) = \pi(\beta_0 + \beta_1 v)$, the expected utility of a student i with ability a_i who provides a level of effort e_i and obtains an evaluation v_i is equal to:

[8] $$E(U_i) = E(W_i) - c(e_i) = \pi[\beta_0 + \beta_1 E(v_i)] - \frac{\gamma e_i^{\,2}}{2} = \pi[\beta_0 + \beta_1(a_i + e_i)] - \frac{\gamma e_i^{\,2}}{2}$$

Students decide the level of effort which maximizes their utility function, taking as given how the market rewards effort.

[9] $$e^* = \frac{\pi\beta_1}{\gamma} \qquad\qquad e^* = \frac{\pi\beta_1}{\gamma} < \frac{\pi_1}{\gamma}$$

Substituting eq. [4] in eq. [9] we obtain:

[10] $$e^* = \frac{\pi}{\gamma}\left[\frac{\sigma_a{}^2}{\sigma_a{}^2 + \sigma_\varepsilon{}^2 + \sigma_\eta{}^2}\right]$$

The optimal level of effort increases when $\sigma_\varepsilon{}^2$ and $\sigma_\eta{}^2$ decrease (the evaluation system is less affected by stochastic variables) since employers, receiving a better signal of students' skills, are willing to pay a higher wage premium on the grade attained at school.

Moreover, if the variance of abilities $\sigma_a{}^2$ is higher, then β_1 increases, positively affecting the effort provided by students. Evaluation is more important when the variability in abilities is higher. In addition, when heterogeneity in individual abilities is high the effect produced by shocks is less relevant and the signal provided by schools is more informative.
In the appendix we show that, assuming $v_i = a_i e_i + \varepsilon + \eta$, low ability students react less compared to high ability students to signal improvements.

The effects of the evaluation system accuracy on student welfare

In this section we evaluate whether students' welfare improves when the evaluation system adopted by the school system becomes more precise.

Considering the optimal level of effort $e^* = \frac{\pi\beta_1}{\gamma}$, the student's expected utility can be written as:

[11]
$$E(U_i) = \pi\left[\beta_0 + \beta_1\left(a_i + \frac{\pi\beta_1}{\gamma}\right)\right] - \frac{(\pi\beta_1)^2}{2\gamma}$$

Deriving the individual expected utility with respect to $\sigma^2 = \sigma_\varepsilon^2 + \sigma_\eta^2$, after some rearrangements, we obtain:

[12]
$$\frac{\partial E(U)}{\partial \sigma^2} = \frac{\partial \beta_1}{\partial \sigma^2}\left[\pi(a_i - \bar{a}) + \left(\frac{\pi^2}{\gamma}\right)(1-\beta_1)\right]$$

Individual utility reduces when σ^2 increases, when:

[13] $$a_i > \tilde{a} = \bar{a} - \left(\frac{\pi}{\gamma}\right)(1-\beta_1) \qquad \text{since } \frac{\partial \beta_1}{\partial \sigma^2} < 0$$

It follows that students whose ability is above the threshold $\tilde{a}$ are negatively affected by less accurate evaluation systems, while students with abilities below this threshold are positively affected.

The threshold value $\tilde{a}$ decreases when productivity increases. In highly productive economic systems also individuals with relatively low abilities prefer more accurate school performance evaluation systems.

Less accurate evaluation systems produce, in fact, two effects: 1) they lead to a more egalitarian pay structure; 2) they reduce effort and, as a consequence, reduce the total output produced in the economy. When π is high the wage reduction deriving from the lower level of effort tends to counterbalance the positive effect that low ability individuals obtain from a low value of β_1.

A comparison between centralized and decentralized evaluation systems

With a centralized evaluation system students are evaluated according to a common standard and $\sigma^2{}_\eta = 0$. On the other hand, we assume that due to very high administration costs, this type of exam is undertaken only at the end of the educational process and shocks affecting students may influence their performance at exams. It follows that the variance of evaluations awarded by the centralized system is equal to $Var(v) = \sigma_a^2 + \sigma_\varepsilon{}^2$.

When evaluation is at decentralized level, delegated to teachers, it is possible to evaluate student performance a large number of times, which we denote with n. Effects deriving from stochastic variables related to student performance are reduced, but different teachers adopt different evaluation methods. The variance of evaluation v_i is equal to: $Var(v) = \sigma_a^2 + \frac{\sigma_\varepsilon{}^2}{n} + \sigma_\eta^2$.

Comparing the expected utility of individual i under a centralized examination system with his expected utility under a decentralized evaluation system, we obtain that for $a_i = \bar{a}$, a centralized evaluation system is preferred when:

[14] $$\left(\beta_1{}^C - \beta_1{}^D\right)\frac{\pi}{\gamma}\left[1 - \frac{\beta_1{}^C + \beta_1{}^D}{2}\right] > 0$$

Since $\beta_1{}^C < 1$ and $\beta_1{}^D < 1$, the term in square brackets is always positive, condition [14] holds when $\beta_1{}^C > \beta_1{}^D$, that is when $\sigma_\eta^2 > \sigma_\varepsilon^2 - \frac{\sigma_\varepsilon^2}{n}$.

The difference $\sigma_\varepsilon^2 - \frac{\sigma_\varepsilon^2}{n}$ increases when σ_ε^2 increases, implying that the advantage of decentralized systems is higher when shocks affecting student performance have a higher variance. Then, these systems may have greater advantages for students enrolled in primary school, since their performance is usually more influenced by emotional factors.

Class size and decentralized evaluation systems

Generally teachers' evaluations are considered costless. Nevertheless, if teachers face very large classes it may result difficult for them to judge students on the basis of daily interactions, participation to work-class, etc. When class size increases the cost of evaluating students increases and teachers may assign grades on the basis of a lower number of evaluations.

The central authority is able to define class size, but it is not able to define the number of evaluations that students have to undertake.

We analyze the choice of the class-size by a policy maker who takes into account teachers' behaviour.

We model this choice as a sequential game in which, in the first stage, the policy-maker sets the class-size, while in the second stage, teachers decide how many evaluations to undertake.

Let us assume a school system with N students, C classes and C teachers, where $S = \frac{N}{C}$ is the size of each class.

Teachers maximize the following utility function: $U^T = E(W)S - nS^2$

[15] $$U^T = \pi\left[\bar{a} + \frac{n\sigma_a^2}{n\sigma_a^2 + \sigma_\varepsilon^2 + n\sigma_\eta^2}\left(\frac{\pi}{\gamma}\right)\right]S - nS^2$$

Deriving [15] with respect to n we obtain the optimal number of evaluations:

[16] $$n = \frac{\left(\sigma_a^2\sigma_\varepsilon^2\frac{\pi^2}{\gamma}\right)^{\frac{1}{2}} - S^{\frac{1}{2}}\sigma_\varepsilon^2}{S^{\frac{1}{2}}\left[\sigma_a^2 + \sigma_\eta^2\right]}$$

n reduces when S increases; increases when σ_ε^2 increases and decreases when σ_η^2 increases.

We analyze the choice of the optimal class size when the policy-maker aims to maximize the wage obtained by students on the labour market net of school costs, *D*, due to the wages paid to teachers and to the rental value of the capital associated to each classroom.

The social welfare function, considering the wage obtained by the representative student with ability $\bar{a}$ net of cost per student, is given by:

[5] $$V = \pi\left[\bar{a} + \left[\frac{\sigma_a^{\ 2}}{\sigma_a^{\ 2} + \dfrac{\sigma_\varepsilon^{\ 2}}{n*(S)} + \sigma_\eta^{\ 2}}\right]\frac{\pi}{\gamma}\right] - \frac{D}{S}$$

Substituting n* in the social welfare function and deriving V with respect to S, we obtain the optimal class size:

[16] $$S = \left[\frac{D}{\pi} \left(\frac{1}{\sigma_a{}^2 \sigma_\varepsilon{}^2 \gamma} \right)^{\frac{1}{2}} 2\left(\sigma_a{}^2 + \sigma_\eta{}^2 \right) \right]^{\frac{2}{3}}$$

When the variance σ_ε^2 increases the optimal class size reduces.

When $\sigma_\eta{}^2$ increases it is optimal to define larger classes since grades awarded by the school system are not a good signal of students abilities and are scarcely rewarded on the labour market.

When the cost of education per student increases the optimal class size increases.

When the productivity of skills is higher it is optimal to reduce class size.

In this analysis, for the sake of simplicity, we have neglected the direct effect of class size on the human capital accumulation process. In a more general framework, considering this aspect, it is possible to show that in educational systems based on decentralized evaluations, the optimal class size is smaller compared to systems based on centralized evaluations.

While in centralized evaluation systems the optimal class size only depends on the marginal benefit deriving from smaller classes in terms of student achievement and marginal costs related to higher expenditures for wages and rental capital, under decentralized evaluation systems class size also affects how informative evaluations are of individual skill.

Larger classes may reduce the frequency of evaluations undertaken by teachers and worsen the informative value of evaluations. Instead, this effect does not play any role in the definition of optimal class size in centralized examination systems.

Concluding Remarks

- We have shown that more precise evaluation systems, being associated to a higher reactivity of wages to school grades, induce an higher level of student effort. Low ability students tend to react less compared to high ability students.

- Whereas high ability individuals strictly prefer more precise evaluation systems, low ability individuals may prefer less precise evaluations.

- When labor productivity increase also individuals with relatively low ability prefer more precise evaluation systems.

- We have used our framework to compare costs and benefits of centralized vs. decentralized evaluations. The performance of the decentralized evaluation system improves when the measurement errors deriving from shocks hitting students are particularly important (for example, when students are very young).

- We have shown that there is a relationship among class size and evaluation systems.