Cyclical Upgrading of Labor and Unemployment Differences across Skill Groups

Andri Chassamboulli*

October 1, 2007

Abstract

This paper examines the labor market cyclical dynamics in a search and matching model which allows for job finding rates to vary endogenously across skill groups. In the economy I examine skilled individuals can perform both skilled and unskilled jobs, whereas unskilled individuals can only perform unskilled jobs. The possibility of on-the-job search induces skilled workers to take transitorily jobs below their skill level, thereby influencing the employment prospects of lower skill groups. As the relative profitability of skilled and unskilled jobs changes over the business cycle, firms respond accordingly by adjusting the skill mix of vacancies, thereby influencing the chances skilled and unskilled workers find jobs unevenly. The model highlights the importance of a vertical type of skill mismatch that takes the form of workers downgrading to lower job levels to escape unemployment and upgrading by on the job search, in explaining why typically skilled unemployment lower and less responsive to business cycles. At the same time, the model is consistent with well established evidence that the quality of job-worker matches and job-to-job transition rates are procyclical.

*Department of Economics, University of Cyprus (e-mail: andricha@ucy.ac.cy). I am greatly indebted to John Haltiwanger, Jeff Smith, John Shea, Michael Pries for the invaluable support and advice and Christopher Pissarides for extensive comments on an earlier version of the paper.

1 Introduction

Average unemployment rates hide dramatic differences across skill groups. As shown in Figure 1, the unemployment rate falls substantially with education. Moreover, it is obvious to the naked eye that the lower the level of education, the higher the rise in the unemployment rate in downturns. The question that follows is why typically the burden of unemployment falls more heavily on the lower educated. Several empirical studies partly explain this fact by showing that while the entry rate into unemployment is lower, the exit rate from unemployment is higher at higher educational attainment.[1] With regard to cyclical changes in unemployment rates, Hall (2005) and Shimer (2005a) shift the attention in job finding rates. Both argue that while job separation rates are almost acyclical, job finding rates are highly procyclical, suggesting that differences in the cyclical behavior of unemployment rates across skill groups are mainly due to differences in the cyclical behavior of job finding rates.

There are good theoretical grounds as to why entry into unemployment declines with education. Skilled workers are more likely to accumulate firm specific capital, making it more difficult for firms to make them redundant.[2] However, hiring skilled individuals is typically more costly, therefore it is less apparent why should the job finding rates of skilled individuals be relatively high and less responsive to business cycles. One argument is that at the prevailing wages, the relative demand for skilled as opposed to unskilled workers remains sufficiently large. But as also emphasized in Nickell (1979), it may be the case that skilled workers manage to keep their employment rates relatively high, by taking jobs below their skill level while searching for more suitable employment instead of searching while unemployed.[3] Naturally, skilled workers qualify for a wider range of job

[1]For evidence that joblessness falls more heavily on the lower skill groups, see e.g., Topel (1993), van Ours and Ridder (1995), Manacorda and Petrongolo (1999), and Juhn *et al.* (2002). For evidence that job finding rates rise with education, see e.g., Topel (1984), Beach and Kaliski (1987), and Petrongolo (2001), and for evidence that separation rates decrease with education, see e.g., Nickell (1979), Royalty (1998), Polsky (1999), Fallick and Fleischman (2001), and Nagypal (2004).

[2]The theoretical relation between turnover and firm specific capital is emphasized in Jovanovic (1979).

[3]There is a variety of reasons skilled workers may be hired for low-skill positions. These include imperfections in the matching process and productivity gains. Similarly, there are several reasons skilled workers may accept transitorily low-skill job offers. As Blau and Robins (1990), and Belzil (1996) show, on-the-job search is more efficient than unemployed search. The former also find that high unemployment and fewer job offers in recessions induce searchers to reject fewer job offers. Moreover, stigma effects associated with long unemployment spells may induce workers to be less selective in matching; see for instance, Vishwanath (1989).

types, thus are relatively more capable of finding transitory employment as opposed to remaining unemployed until a suitable job offer comes along.

In this paper I develop a search and matching model to investigate the importance of on-the-job search at the higher end of the skill distribution, in explaining the observed differences in the cyclical responsiveness of unemployment rates across skill groups. In the economy I examine firms open vacancies for either high-productivity jobs, which have high skill requirements, or low-productivity jobs, with lower skill requirements. High-skill workers are best suited for high-skill jobs, but they also qualify for low-skill jobs, whereas, low-skill workers qualify only for low-skill jobs. To capture matching imperfections, I assume that workers cannot ex-ante identify the types of vacancies, thus cannot target only the jobs they are suited for. Consequently, the job finding rate of low-skill workers is proportional to the share of low-skill vacancies in the total number of vacancies posted. However, the skill mix of vacancies does not influence high-skill workers' job finding rates. This is because the possibility of on-the-job search induces them to accept the low-skill jobs they encounter, and continue searching while employed for high-skill jobs. Cyclical changes in the skill composition of new employment opportunities affect the job finding rates of the two types of workers unevenly, causing the skill composition of job seekers to change. In turn, cyclical changes in the composition of job seekers affect the effective matching rates of firms with low- and high-skill vacancies unevenly, causing the skill composition of the vacancies opened to change, resulting in a circle of interactions.

In this framework, when high-skill workers accept transitorily low-skill jobs, they influence the profits of low-skill jobs in two countervailing ways. On the one hand, since they are likely to abandon low-skill jobs sooner, by crowding out low-skill workers from low-skill positions, they lower the profits of low-skill jobs, and discourage firms from opening low-skill vacancies. On the other hand, when high-skill workers join the queues for low-skill jobs, firms with low-skill vacancies can fill their vacancies faster, thus face lower the recruitment costs. This in turn encourages firms to open more low-skill vacancies. Whether the chances low-skill workers find jobs improve or worsen when high-skill workers accept transitorily low-skill jobs, depends on which of the two effects dominates.

The cyclical pattern in the matching behavior of skilled workers this paper refers to, which takes the form of downgrading to lower job levels to escape unemployment and upgrading by on-the-job search, is supported by several empirical observations. Evidence by Bowlus (1995) and Davis *et al.* (1996), among others, that jobs created in recessions are of lower duration and offer lower wages than jobs created in booms, suggest that the quality of job-worker matches falls in periods of high unemployment. Moreover, the importance and the procyclical behavior of job-to-job transition rates has been emphasized in a number of empirical studies, but recently, Nagypal (2004) documents in addition that both the share of separations accounted by job-to-job flows, and the share of quits that lead to a direct transition into a new job rises with education.[4] The finding of Pissarides and Wadsworth (1994) that the propensity to search on the job is higher among the more educated also supports the view that more educated individuals are prone to search while employed as opposed to unemployed. Finally, there is direct evidence of over-education phenomena at high levels of educational attainment. As documented in Hecker (1992, 1995), since the early 1980's, between 17 and 18 percent of college graduates in the U.S. were employed in jobs that do not require a degree.[5]

I calibrate the model to the U.S. labor market, assuming that the high-skill type refers to college graduates and the low-skill type to individuals with less than college education. The calibration accounts for the much lower separation rate of college graduates and much higher productivity gains associated with hiring college graduates, as reflected in the observed large wage premium for college graduates. Still, the simulations reveal that the so much lower unemployment rate of college graduates can be explained only if a substantial fraction of them is being underemployed in jobs that require less than college education. The calibration yields that on average, 17.8% of college graduates are over-educated, well in line with the empirical graduate over-education measures reported above. I also investigate the possibility that eliminating matching imperfections would yield lower unemployment rates for college graduates, simply because the market for college graduates is tighter due to larger productivity gains associated with skilled positions. I simulate the model assuming that workers can target only the jobs they are best suited for (i.e., search is directed). However, to match the observed unemployment differences, the model with directed search requires unrealistically high wage premiums for college graduates. Unless the nature of matching or wage setting differs significantly across the two sectors, this also suggests that employment at higher educational attainment remains high, due to temporary over-education and upgrading by on-the-job search.

[4]Similarly, Polsky (1999) shows that the negative correlation between educational attainment and separations becomes less apparent when accounting also for quits instead of only layoffs.

[5]Graduate over-education measures of the same range can also be found for many European countries. For instance, Green *et al.* (1999) find that just over 20% of graduates in the UK are genuinely over-educated for their jobs, and Oliver and Raymond (2003) show that in 1998 the proportion of over-educated college graduates in Spain was 21%.

In addition, with 17.8% on average of college graduates being over-educated, the cyclical behavior of unemployment rates in the model matches quite well the observed differences in the cyclical behavior of unemployment rates between college graduates and individuals with less than college education. In the model the burden of recessions falls more heavily on the lower educated for two reasons. First, firms respond to a fall in aggregate productivity by shifting the skill mix of vacancies towards the more productive type, which is high-skill in the model. Consequently, both types of workers suffer reductions in their chances of finding jobs, as firms open fewer vacancies per job seeker, but low-skill workers are hurt the most. Second, a higher number of high-skill unemployed resorts to temporary employment in low-skill jobs in recessions, instead of remaining unemployed, while upgrades to high-skill jobs happen more frequently in booms, when job finding rates rise.

After establishing that transitory over-education is crucial in explaining why skilled unemployment remains relative low at all states of the business cycle, the question I ask is how this behavior affects the chances low-skill workers find jobs, and the economy overall. I find that it actually improves the chances lower educated workers find jobs. In the calibrated model, the negative crowding out effect that lowers the average quality of low-skill jobs is small relative to the positive impact of lower recruitment costs for firms with low-skill vacancies. Moreover, when high-skill workers accept low-skill jobs, the pool of potential hires is higher at both segments of the labor market. Not only firms with low-skill vacancies benefit from high-skill unemployed joining the queues for low-skill jobs, but also firms with high-skill vacancies benefit from the presence of on-the-job searchers. The resulting higher recruitment activity at both segments of the labor market, maintains a higher incentive for firms to open vacancies in both sectors. Hence, both high- and low-skill employment is higher when high-skill workers accept transitorily low-skill jobs. In addition, high-skill employment is higher not only because of the on-the-job searchers, but also because the number of suitably matched high-skill workers is higher. Finally, I find that both sectors exhibit more cyclical employment growth when high-skill workers accept transitorily low-skill jobs, but especially the high-skill sector. This is because with on-the-job searchers, potential hires for firms with high-skill vacancies capture a higher share in the pool of job seekers in booms, when there are fewer unemployed job seekers, and vice versa in recessions.

Although this paper is not the first to account for the asymmetric nature of matching, it is the first to explore the cyclical dynamics of the labor market in a model that allows for job finding rates to vary endogenously across skill groups. Albrecht and Vroman (2002), Gautier (2002), and Dolado *et. al.* (2004), also assume that skilled workers can perform both skilled and unskilled jobs, whereas unskilled workers can perform only unskilled jobs. By incorporating asymmetric matching, these studies link over-education phenomena to skill-biased technological shocks and the crowding out of workers at the lower segment of the labor market. Their focus, however, is to explain long-run uneven developments in the unemployment rates of different skill groups. Therefore, by focusing only on steady states, little attention has been paid to the cyclical implications of transitory over-education and upgrading by on-the-job search. By incorporating job heterogeneity and on-the-job the cyclical upgrading of labor has been emphasized in Barlevy (2002) and more recently in Krause and Lubik (2006). The view formalized in Barlevy is that recessions impede the reallocation of workers from less productive to more productive jobs, because firms post fewer vacancies per job seeker. Similarly, Krause and Lubik highlight the role of on-the-job search in explaining cyclical changes in the composition of new employment opportunities, and observed worker mobility patterns. The model in Krause and Lubik does not feature worker heterogeneity, while Barlevy's model accounts for worker heterogeneity, but as the author argues, in order to make the role of aggregate shocks more transparent, employs a symmetric framework, in which matching rates are equal across skill categories. Hence, existing models offer a characterization of the cyclical behavior of worker flows only in terms of average or representative values and overlook the observed salient differences across skill groups. More importantly, they overlook the across-skill externalities that arise, when workers of different skill compete for the same types of jobs.

The rest of the paper is organized as follows. Section 2 outlines the dynamic model in which aggregate productivity fluctuates over time. Section 3 defines a steady state equilibrium and uses analytic results to provide a more rigorous intuition for the results of the dynamic model that follow. In section 4, I calibrate and numerically solve the dynamic model outlined in section 2, and discuss its qualitative and quantitative implications. Section 4 also studies the consequences of transitory over-education on low-skill employability and its implications for the cyclical dynamics of the labor market overall. In section 5, I examine the properties of the model in which search is directed. Finally, section 6 concludes.

2 The Model

2.1 The Labor Market

The labor force is composed by two types of risk neutral workers: a fraction δ is low-skill (l) and the remaining $(1-\delta)$ is high-skill (h). Similarly, vacancies can be either high-skill

(h) or low-skill (l), but the mix is determined endogenously. High-skill workers can perform both types of jobs, whereas low-skill workers can only perform low-skill jobs. Accordingly, a low-skill worker can be either employed and producing in a low-skill job or unemployed and searching, while a high-skill worker can be in any if the following three states: employed and producing in a high-skill job, unemployed and searching, and employed and producing in a low-skill job, but simultaneously searching for a high-skill job. I label a worker in the latter state as over-qualified job seeker.

Each firm has at most one job, which can be either vacant and searching for candidates or filled and producing. The mass of each type of vacancy is determined endogenously by a free-entry condition. The exogenous component of job destruction follows a Poisson process with arrival rate s_i, where $i = (h, l)$, and is assumed to be specific to the type of worker. Whenever a match is destroyed the job becomes vacant and bears a maintenance cost c_l, specific to its type.

Wages are chosen to divide the surplus of the match between the worker and the firm in fixed proportions, in line with Nash bargaining. With γ being the workers' bargaining power, a share γ of the surplus goes to the worker, while the rest $1 - \gamma$ goes to the firm. When unemployed the worker enjoys a productivity flow b_i, which can be interpreted as the opportunity cost of working.[6]

2.1.1 Match Productivities

The productivity of each job-worker match is assumed to be the product of a stochastic aggregate component y, and a match specific component α_{ij}, when a worker of type $i = (h, l)$ is matched with a job of type $j = (h, l)$. The aggregate component is assumed to follow a discrete-state Markov process. The vector of possible aggregate productivity realizations is given by $\bar{y}$ and the elements of the transition matrix Π are given by $\pi_{nm} = prob\{y_{t+1} = \bar{y}_m \setminus y_t = \bar{y}_n\}$. The condition that ensures a match is formed in equilibrium is simply that the productivity of the match is higher than the worker's opportunity cost of working, i.e., $y\alpha_{ij} > b_i$. This condition ensures that the surplus of the job is positive, and the wage it offers is higher than the opportunity cost of working. Since low-skill workers do not have the minimum required skills to perform high-skill jobs, the underlying assumption is that

[6]Since there is no government or any form of taxation in the model, I avoid naming b_i as unemployment benefit. In reality, the unemployment benefit is only one of the factors that determine b_i. A variety of additional factors could influence a worker's opportunity cost of working, including the value attributed to leisure, spousal income, and the value of home production.

$y\alpha_{lh} - b_l \leq 0$, which implies that when filled by a low-skill worker, high-skill jobs generate losses instead of a surplus.

I assume that high-skill workers are best suited for high-skill jobs and are therefore more productive when matched with high- instead of low-skill jobs. This implies that $y\alpha_{hh} - b_h > y\alpha_{hl} - b_h$ and ensures that over-qualified workers receive lower wages than suitably matched workers, because the former generate lower surplus. Hence, under this assumption, over-qualified workers have a natural incentive to search on the job for high-skill jobs. Moreover, I assume for convenience that the rate at which workers meet high-skill vacancies is the same regardless of whether the worker is employed or not. Consequently, as long as $y\alpha_{hl} > b_h$, so that the surplus of a low-skill job filled by an over-qualified worker is positive, it is optimal for unemployed high-skill workers to accept transitorily low-skill jobs, since they retain their chances of finding a high-skill job by continuing their search while employed.

Finally, I assume that the net productivity of a suitably matched low-skill worker is at least as high as the net productivity of an over-qualified worker. That is, $y\alpha_{ll}-b_l \geq y\alpha_{hl}-b_h$. This is to ensure without any additional restrictions in the parameter space that firms with low-skill vacancies are better off hiring low-skill instead of over-qualified workers. Since the latter are more likely to quit, low-skill positions always generate higher profits when filled by suitable instead of over-qualified workers. Note that this assumption does not necessarily imply that over-qualified workers are less productive than suitably matched low-skill workers. Even if $\alpha_{hl} > \alpha_{ll}$, this condition is still satisfied when high-skill workers' opportunity cost of working is sufficiently higher than that of low-skill workers.[7]

2.1.2 Matching and Timing

Firms and workers meet each other via a matching technology $m(\nu_t, z_t)$, where $\nu_t = \nu_t^h + \nu_t^l$ is the number of high- and low-skill vacancies, and $z_t = u_t^h + u_t^l + e_t^{hl}(1 - s_h)$ is the number of job seekers; u_t^h and u_t^l denote the number of high- and low-skill unemployed, and $e_t^{hl}(1 - s_h)$ the number of over-qualified workers who survive separation, and thus continue searching on the job. The function $m(\cdot, \cdot)$ is strictly increasing in its arguments, and exhibits constant returns to scale. This allows me to write the flow rate at which workers meet vacancies as $m(\theta_t)$, where $\theta_t = \frac{\nu_t^h+\nu_t^l}{u_t^h+u_t^l+e_t^{hl}(1-s_h)}$ captures the degree of labor

[7] In section 4 where I calibrate the model, I choose to let the data guide me on my choice of parameter values for α_{hh} α_{hl}, α_{ll}, b_h and b_h, instead of imposing ex-ante hypothetical restrictions. Nevertheless, the calibrated values conform to these assumptions.

market tightness.

I assume that workers cannot ex-ante distinguish the vacancy type, and thus cannot direct their search towards a specific type of vacancy. I make this assumption to capture the notion that matching is imperfect: workers do not always arrive at the jobs they are best suited for. Consequently, low-skill workers encounter low-skill vacancies with probability per unit of time that is proportional to the fraction of low-skill vacancies. Similarly, high-skill workers encounter low- and high-skill vacancies with a probability per unit of time that is proportional to the fraction of low- and high-skill vacancies, respectively. Assuming that $\eta_t = \frac{\nu_t^l}{\nu_t^l+\nu_t^h}$, the effective matching rate of low-skill workers is $\eta_t m(\theta_t)$, while over-qualified workers relocate into high-skill jobs at rate $(1-\eta_t)m(\theta_t)$. Unemployed high-skill workers accept both high- and low-skill jobs, thus their effective matching rate is $m(\theta_t)$.

The timing within a period is as follows. At the beginning of the period, the realization of aggregate productivity is revealed and agents produce. After agents produce, some of the existing matches are exogenously destroyed. Subsequently, firms post vacancies to ensure zero profits. Finally, search takes place. Based on the matching rates specified above, some over-qualified workers quit to high-skill jobs, while some unemployed workers find jobs. Letting $e_t = \left\{e_t^{hh}, e_t^{hl}, e_t^{ll}\right\}$ be the distribution of employed workers across types of matches at the beginning of period t, the distribution of employed workers at the beginning of period $t+1$ is given by:

$$\begin{aligned} e_{t+1}^{ll} &= e_t^{ll}(1-s_l) + \eta_t m(\theta_t)\left[\delta - e_t^{ll}(1-s_l)\right] \\ e_{t+1}^{hh} &= e_t^{hh}(1-s_h) + (1-\eta_t)m(\theta_t)\left[1-\delta-e_t^{hh}(1-s_h)\right] \\ e_{t+1}^{hl} &= e_t^{hl}(1-s_h) + \eta_t m(\theta_t)\left[1-\delta-(e_t^{hl}+e_t^{hh})(1-s_h)\right] \\ &\quad -(1-\eta_t)m(\theta_t)e_t^{hl}(1-s_h) \end{aligned} \tag{1}$$

I now turn to the effective matching rates of firms. The rate at which a firm meets a job seeker of any type is equal to $q(\theta_t) = m(1, \frac{1}{\theta_t})$, which is decreasing in θ_t and exhibits the standard properties: $lim_{\theta_t\to 0}^{q(\theta_t)} = lim_{\theta_t\to\infty}^{\theta_t q(\theta_t)} = \infty$, and $lim_{\theta_t\to\infty}^{q(\theta_t)} = lim_{\theta_t\to 0}^{\theta_t q(\theta_t)} = 0$. Low-skill vacancies match only with unemployed job seekers. This is because an over-qualified worker has no incentive to change employer unless the new employer offers a high-skill job. Accordingly, sometimes firms with low-skill vacancies meet over-qualified workers who refuse to match. It follows that low-skill vacancies match with low-skill workers at rate $\psi_t\varphi_t q(\theta_t)$, and with high-skill workers at rate $\psi_t\left(1-\varphi_t\right)q(\theta_t)$, where $\varphi_t = \frac{u_t^l}{u_t^l+u_t^h}$ and $\psi_t = \frac{u_t^l+u_t^h}{u_t^l+u_t^h+e_t^{hl}(1-s_h)}$. Likewise, employers with high-skill vacancies do not hire the low-skill workers they meet. Consequently, high-skill vacancies match only with either over-qualified or unemployed high-skill workers, and thus, their effective matching rate is $(1 - \psi_t \varphi_t)\, q(\theta_t)$.

2.2 Value Functions

To describe the value functions I adopt the following notation. For the worker, U_t^i is the value of being unemployed, and W_t^{ij} is the value of being employed. For the firm, V_t^j is the value of a vacancy, and J_t^{ij} is the value of a filled job. In all cases, i denotes the type of worker and j the type of job. Moreover, in what follows $\beta = \frac{1}{1+r}$ is the discount factor.

2.2.1 Workers

A low-skill worker's value of being unemployed satisfies

$$U_t^l = b_l + \beta E_t \left[\eta_t m\left(\theta_t\right) W_{t+1}^{ll} + \left(1 - \eta_t m(\theta_t)\right) U_{t+1}^l \right] \tag{2}$$

The interpretation is straightforward. The value of being unemployed is equal to the payoff of being unemployed in the current period, b_l, plus the present value of the expected payoff next period. The latter is given by the probability the worker finds a low-skill job, $\eta_t m(\theta_t)$, times the value of having a low-skill job next period, W_{t+1}^{ll}, plus the probability the worker remains unemployed, $(1 - \eta_t m(\theta_t))$, times the value of being unemployed in the next period, U_{t+1}^l. The expectations operator E_t depends on the transition matrix of aggregate productivity Π, and the transition equations described in (1).

The rest of the value functions take a similar form. Given that high-skill workers accept both types of jobs, the value of unemployment to a high-skill worker satisfies

$$U_t^h = b_h + \beta E_t \left[\; m(\theta_t)[\eta_t W_{t+1}^{hl} + (1 - \eta_t) W_{t+1}^{hh}] + (1 - m(\theta_t)) U_{t+1}^h \; \right] \tag{3}$$

The values of being suitably matched satisfy

$$W_t^{hh} = w_t^{hh} + \beta E_t \left[s_h U_{t+1}^h + (1 - s_h) W_{t+1}^{hh} \right] \tag{4}$$

$$W_t^{ll} = w_t^{ll} + \beta E_t \left[s_l U_{t+1}^l + (1 - s_l) W_{t+1}^{ll} \right] \tag{5}$$

while the value of being over-qualified is given by

$$W_t^{hl} = w_t^{hl} + \beta E_t \left[\; s_h U_{t+1}^h + (1 - s_h) W_{t+1}^{hl} + (1 - s_h)(1 - \eta_t) m(\theta_t) [W_{t+1}^{hh} - W_{t+1}^{hl}] \; \right] \tag{6}$$

where w_t^{ij} denotes the wage rate in each case. The value of being over-qualified incorporates in addition the expected gain from on-the-job search. This is given by the last term in the bracket, which is interpreted as follows: given that the match survives job destruction with a probability $(1-s_h)$, the worker meets a high-skill vacancy with a probability $(1-\eta_t)m(\theta_t)$, and obtains a surplus $[W_{t+1}^{hh} - W_{t+1}^{hl}]$ from switching jobs.

2.2.2 Firms

For the firms, the asset values of filling low- and high-skill vacancies with suitable workers are given, respectively, by

$$J_t^{hh} = y\alpha_{hh} - w_t^{hh} + \beta E_t \left[s_h V_{t+1}^h + (1-s_h) J_{t+1}^{hh} \right] \tag{7}$$

$$J_t^{ll} = y\alpha_{ll} - w_t^{ll} + \beta E_t \left[s_l V_{t+1}^l + (1-s_l) J_{t+1}^{ll} \right] \tag{8}$$

The value of filling a low-skill vacancy with an over-qualified worker is given by

$$J_t^{hl} = y\alpha_{hl} - w_t^{hl} + \beta E_t \left[\ s_h V_{t+1}^l + (1-s_h) J_{t+1}^{hl} - (1-s_h)(1-\eta_t) m(\theta_t) [J_{t+1}^{hl} - V_{t+1}^l] \ \right] \tag{9}$$

It incorporates in addition the loss due to endogenous quits. This is captured by the last term in the bracket. If the job is not exogenously destroyed, the worker continues searching on the job, and quits with a probability $(1-\eta_t)m(\theta_t)$, in which case the job becomes vacant. Finally, the values of opening high- and low-skill vacancies are given by

$$V_t^h = -c_h + \beta q(\theta_t) E_t \left[(1-\psi_t \varphi_t) J_{t+1}^{hh} + \psi_t \varphi_t V_{t+1}^h \right] \tag{10}$$

$$V_t^l = -c_l + \beta q(\theta_t) E_t \left[\ \psi_t \varphi_t J_{t+1}^{ll} + \psi_t (1-\varphi_t) J_{t+1}^{hl} + (1-\psi_t) V_{t+1}^l \ \right] \tag{11}$$

2.2.3 Surpluses

Given that the worker and the firm share the surplus in fixed proportions with γ being the worker's share, the wage w_t^{ij} satisfies the following Nash bargaining conditions

$$\begin{aligned} W_t^{ij} - U_t^j &= \gamma S_t^{ij} \\ J_t^{ij} - V_t^i &= (1-\gamma) S_t^{ij}. \end{aligned} \tag{12}$$

where S_t^{ij} denotes the surplus of the match, defined as

$$S_t^{ij} \equiv W_t^{ij} + J_t^{ij} - U_t^i - V_t^j \tag{13}$$

Substituting the value functions together with the Nash bargaining conditions in (12), into the surplus expression above yields

$$S_t^{ll} = y\alpha_{ll} - b_l + \beta E_t [(1-s_l) S_{t+1}^{ll} - \eta_t m(\theta_t) \gamma S_{t+1}^{ll}] \tag{14}$$

$$S_t^{hh} = y\alpha_{hh} - b_h + \beta E_t \left[(1-s_h)S_{t+1}^{hh} - (1-\eta_t)m(\theta_t)\gamma S_{t+1}^{hh} - \eta_t m(\theta_t)\gamma S_{t+1}^{hl} \right] \tag{15}$$

$$S_t^{hl} = y\alpha_{hl} - b_h + \beta E_t \left[\begin{array}{c} (1-s_h)S_{t+1}^{hl} - (1-\eta_t)m(\theta_t)\gamma S_{t+1}^{hh} - \eta_t m(\theta_t)\gamma S_{t+1}^{hl} \\ -(1-s_h)(1-\eta_t)m(\theta_t)[S_{t+1}^{hl} - \gamma S_{t+1}^{hh}] \end{array} \right] \tag{16}$$

The surplus of a low-skill job filled by a low-skill worker, S_t^{ll}, takes the standard form. The term outside of the bracket gives the match productivity net of the opportunity cost of working. The first term in the bracket gives the surplus given that the match survives to the next period, and the second term in the bracket the loss to the worker for giving up searching for a job while unemployed. Once employed the worker gives up the opportunity to match with a low-skill vacancy with a probability $\eta_t m(\theta_t)$ and gain a share γ of the resulting surplus S_{t+1}^{ll}. Therefore, the value of this opportunity is subtracted from the surplus. As regards the surplus of a high-skill job, S_t^{hh}, the only difference is that when suitably matched, high-skill workers give up searching for both high- and low-skill jobs. Consequently, the additional term $(1-\eta_t)m(\theta_t)\gamma S_{t+1}^{hh}$, which reflects the value of the opportunity to match with a high-skill vacancy is also subtracted from the surplus. The surplus of a low-skill job filled by an over-qualified worker, S_t^{hl}, takes also into account the cost of endogenous quits, which is given by the last term in the bracket. Given that the match survives to the next period, with a probability $(1-\eta_t)m(\theta_t)$, the over-qualified worker quits to a high-skill job, in which case the worker obtains a share γ of S_{t+1}^{hh}, but S_{t+1}^{hl} is lost.

It is easy to verify just by looking at the surplus expressions above, that as in the standard model, an increase in the meeting rate $m(\theta_t)$ lowers the surpluses of all jobs. This is because a higher meeting rate raises workers' value of searching while unemployed. As firms need to compensate the workers they hire for giving up searching, this in turn lowers the surplus of filled jobs. It is also straightforward to verify that as long as $S_{t+1}^{hh} \geq S_{t+1}^{hl}$, so that low-skill jobs offer lower wages to high-skill workers than high-skill jobs, upgrading the skill composition of vacancies (i.e., lowering η_t), raises the surplus of low-skill jobs, S_t^{ll}, but lowers the surplus of high-skill jobs, S_t^{hh}. The intuition is similar; when high-skill vacancies are relatively more abundant, high-skill workers can more easily find high-skill jobs, which offer higher wages, and avoid temporary employment in low-skill jobs, while low-skill workers have more difficulty finding jobs. This in turn, raises the value of searching while unemployed for high-skill workers, and lowers it for low-skill workers. The surpluses of high- and low-skill jobs change accordingly.

The impact of a fall in η_t on S_t^{hl} is more cumbersome to determine. On the one hand, with high-skill vacancies relatively more abundant, over-qualified workers can more easily upgrade to high-skill jobs. Hence, as endogenous quits are more likely, the surplus declines. On the other hand, the value of the forgone opportunity to match with a low-skill vacancy while unemployed, which is subtracted from the surplus, is lower when η_t is lower. The overall impact on the surplus depends on which of the two effects dominates, making it difficult to establish it analytically.

2.3 Equilibrium

Given free entry, $V_t^i = 0$ should be satisfied in equilibrium. Therefore, $E_t V_t^i = 0$ must also hold in equilibrium. Applying these conditions to (10) and (11) together with the Nash bargaining conditions in (12) yields the following free-entry conditions for low- and high-skill vacancies, respectively

$$(1-\gamma)\beta E_t\left[\psi_t\varphi_t S_{t+1}^{ll} + \psi_t(1-\varphi_t)S_{t+1}^{hl}\right] = \frac{c_l}{q(\theta_t)} \tag{17}$$

$$(1-\gamma)\beta E_t[(1-\psi_t\varphi_t)S_{t+1}^{hh}] = \frac{c_h}{q(\theta_t)} \tag{18}$$

The free-entry conditions are such that in equilibrium the expected profit from filling a vacancy (left hand side) is equal to the costs of keeping the vacancy unfilled (right hand side), and implicitly define θ_t and η_t.

More formally, the equilibrium is given by a vector $\{\theta, \eta\}$ that for each realization of aggregate state, y, and distribution of employment, $e_t = \left\{e_t^{hh}, e_t^{hl}, e_t^{ll}\right\}$, satisfies the following: (i) the three types of matches are formed voluntarily, i.e., $y\alpha_{hh} > b_h$, $y\alpha_{ll} > b_l$, and $y\alpha_{lh} > b_h$; (ii) the two free entry conditions in (17) and (18) are satisfied so that the values of maintaining low- and high-skill vacancies are zero; and (iii) the state variables e_t^{hh}, e_t^{hl}, and e_t^{ll} are determined by the set of flow equations in (1). With the characterization of the equilibrium I complete the description of the model.

Before digging deeper into the model a few words are in line regarding the properties of the equilibrium. First, notice that uniform changes in the expected profits of both types of vacancies require offsetting changes in market tightens, θ_t, while unequal changes in the expected profits of high- and low-skill vacancies require adjustments in the equilibrium value of η_t (i.e., adjustments in the skill mix of vacancies) to keep the values of both types of vacancies equal to zero. To establish analytically the intuitive notion that firms respond by shifting the vacancy mix towards high-skill vacancies when the expected surplus of high-skill jobs increases relative to the expected surplus of low-skill jobs, one has to prove that η_t lowers the expected surplus of low-skill vacancies, as captured by the left-hand-side of (17). However, this is not an easy task. As mentioned earlier, although an increase in η_t lowers

S_t^{ll}, the impact on S_t^{hl} can go either way. The illustrative steady-state exercise that follows specifies parameter restrictions, which ensure that an increase in η_t lowers S_t^{hl}. Moreover, the simulations of the calibrated dynamic model that follow, confirm this result.

Observe also that unlike the standard model, shifts in the skill composition of job seekers affect the two sectors unevenly, thus altering the skill composition of vacancies opened. A reduction in the fraction of unemployed job seekers, ψ_t, (or equivalently, an increase in the fraction of over-qualified job seekers) lowers the expected surplus of low-skill jobs, while it raises the expected surplus of high-skill jobs. Assuming that the equilibrium value of η_t declines when the expected surplus of low-skill jobs declines relative to the expected surplus of high-skill jobs, it follows that an increase in the fraction of over-qualified job seekers induces firms to open relatively more high-skill vacancies, making it more difficult for low-skill workers to find suitable jobs. Moreover, when $S_{t+1}^{ll} - S_{t+1}^{hl} > 0$, it can be easily verified by rearranging terms in (17) that an increase in the fraction of high-skill job seekers (i.e., a reduction in $\psi_t \phi_t$), lowers the expected surplus of low-skill vacancies, but raises the expected surplus of high-skill vacancies. Hence, when low-skill jobs generate a higher surplus when filled by low-skill instead of over-qualified workers, a rise in the fraction of high-skill job seekers discourages firms from opening low-skill vacancies.

3 Steady State

In this section, I first solve for a unique steady state equilibrium and then, I illustrate the impact of a permanent decline in aggregate productivity y on market tightness θ and skill composition of vacancies as captured by η. The proofs of the results presented in this section are given in the Appendix. The purpose of this analytic exercise is to provide a more rigorous intuition for the results of the numerical analysis that follow. Evidently, this exercise is limited, because it does not provide insights into the dynamic associated with shocks. The task of characterizing the dynamic responses of variables to temporary shocks is taken up in subsequent sections.

To keep calculations tractable, I consider the case $s_h = s_l = s$, $b_h = b_l = b$, and $\alpha_{hl} = \alpha_{ll}$. This choice of parameters ensures that over-qualified workers have an incentive to search on the job, because high-skill jobs offer better wages (i.e., $S^{hh} \geq S^{hl}$), and that firms with low-skill vacancies are better off hiring low- as opposed to high-skill workers (i.e., $S^{ll} \geq S^{hl}$).

Assuming continuous time, the steady state free entry conditions along which the value of opening a vacancy is equal to zero, are given by the set of equations below.

$$(1-\gamma)[\psi\varphi S^{ll} + \psi(1-\varphi)S^{hl}] = \frac{c_l}{q(\theta)} \tag{19}$$

$$(1-\gamma)(1-\psi\varphi)S^{hh} = \frac{c_h}{q(\theta)} \tag{20}$$

where

$$S^{ll} = \frac{y\alpha_{ll} - b}{(r+s+\gamma\eta m(\theta))} \tag{21}$$

$$S^{hl} = \frac{y\alpha_{ll} - b}{(r+s+\gamma\eta m(\theta) + (1-\eta)m(\theta))} \tag{22}$$

$$S^{hh} = \frac{(y\alpha_{hh} - b)}{(r+s+\gamma(1-\eta)m(\theta))} - \gamma\eta m(\theta)S^{hl} \tag{23}$$

Sufficient parameter restrictions to ensure the steady state equilibrium is unique are: i) $\frac{(y\alpha_{ll}-b)}{(y\alpha_{hh}-b)}\left[\frac{\delta}{(1-\delta)} + \frac{2\gamma}{\gamma+1}\right] \geq \frac{c_l}{c_h}$; ii) $\gamma \geq \frac{1}{2}$ and $\delta \geq \frac{1}{2}$; iii) $\frac{(y\alpha_{ll}-b)}{(y\alpha_{hh}-b)} \leq \gamma$. The first condition ensures that the fraction of low-skill job seekers, $\psi\phi$, decreases when θ increases, and is sufficient to establish that the value of low-skill vacancies declines with θ. Conditions *ii)* and *iii)* ensure that a higher η increases the surplus of low-skill vacancies (left-hand-side of (19)), but lowers the surplus of high-skill vacancies (left-hand-side of (20)). Therefore, if for some exogenous reason the surplus of high-skill jobs increases relative to the surplus of low-skill jobs, η must decline, for the free-entry conditions to be satisfied in equilibrium. Under these conditions, the free-entry conditions (19) and (20) have opposite slopes in the $[\eta, \theta]$ plane, and the equilibrium is characterized by the intersection of the two loci as shown in Figure 2.

A reduction in y lowers the surpluses of both types of jobs. Therefore, both loci shift down in response to a fall in y, and the equilibrium value of θ declines. Intuitively, when aggregate productivity is low, each job is proportionally less productive, thus firms post fewer vacancies per job seeker. The impact on η depends on which of the two types of jobs is hurt the most. In other words, it depends on which of the two loci shifts down by more. To determine this, I first take the ratio of the low-skill free-entry condition to the high-skill free-entry condition, and then evaluate its derivative with respect to y. The ratio is given by,

$$\frac{\psi\varphi}{(1-\psi\varphi)}\frac{S^{ll}}{S^{hh}} + \frac{\psi(1-\varphi)}{(1-\psi\varphi)}\frac{S^{hl}}{S^{hh}} = \frac{c_l}{c_h} \tag{24}$$

After substituting for the surpluses given in expressions (21) to (23), the derivative of this ratio with respect to y is given by,

$$\frac{\partial R}{\partial y} = \left[\frac{b\lambda_3\lambda_1(\psi\varphi\lambda_3 + \psi(1-\varphi)\lambda_2)}{(1-\psi\varphi)\lambda_2[(y\alpha_{hh}-b)\lambda_3 - \gamma\eta m(\theta)(y\alpha_{ll}-b)]^2}\right](\alpha_{hh} - \alpha_{ll}) \tag{25}$$

where $\lambda_1 = [\delta(s + (1-\eta)m(\theta)) + (1-\delta)(s + \eta m(\theta))]$, $\lambda_2 = (r + s + \gamma\eta m(\theta) + (1-\eta)m(\theta))$, and $\lambda_3 = (r + s + \gamma(1-\eta)m(\theta))$. As long as $(a_{hh} - a_{ll}) \geq 0$, the above derivative is positive. This implies that a reduction in aggregate productivity has a stronger negative impact on the value of low-skill vacancies. Therefore, the free-entry condition for low-skill vacancies shifts down relatively more so that both η and θ decline, as illustrated in Figure 3. The reason low-skill jobs are hurt the most is simply that the net productivity of low-skill jobs, $(y\alpha_{ll} - b)$, is lower than the net productivity of high-skill jobs, $(y\alpha_{hh} - b)$. As Shimer (2005b), Hagedorn and Manovskii (2007), and Pries (2007) also point out, the surplus of a match is more sensitive to changes in aggregate productivity when the productivity of the match net of the opportunity cost of employment is small. In the present model, at lower values of y, the percentage gap between the productivity of the job and the opportunity cost of employment declines more for low- than for high-skill jobs, pushing the relative surplus of high-skill jobs up.[8]

Consequently, the burden of a permanent reduction in aggregate productivity falls more heavily on low-skill workers. The reduction in θ implies that high-skill workers also have more difficulty finding vacancies, because $m(\theta)$ declines. However, in addition to the reduction in $m(\theta)$, low-skill workers bear the reduction in η. Hence, they suffer a higher reduction in their matching rate relatively to high-skill workers, implying a relatively higher increase in low-skill unemployment in recessions.

A conclusion regarding the impact of a fall in y on the number of over-qualified high-skill workers cannot be reached based on this analytic result alone. A fall in y implies that high-skill workers encounter low-skill vacancies less frequently, as $\eta m(\theta)$ declines. However, if the rise in high-skill unemployment due to the fall in $m(\theta)$ is sufficiently high, then the number of over-qualified workers may still rise.

For now it is enough to note that aggregate shocks have uneven consequences on the two types of workers. Firms face the choice of which type of vacancy to open and how many vacancies to open. As the relative surplus of the two types of jobs changes when aggregate productivity falls, firms respond not only by lowering the number of vacancies

[8] It is important to point out that this result is not sensitive to the assumption the aggregate productivity shock is multiplicative. An additive aggregate productivity shock (i.e., $y + a_{ij}$ instead of $y\alpha_{ij}$) would imply an even higher increase in the net productivity of high-skill jobs relative to the net productivity of low-skill jobs. Moreover, this result is not sensitive to the assumption that b is the same for both types of workers. Assuming that high-skill workers generate b_h while unemployed and low-skill workers generate b_l while unemployed, the same result would still hold as long as the net productivity of high-skill jobs $(y\alpha_{hh} - b_h)$ is greater than the net productivity of low-skill jobs $(y\alpha_{ll} - b_l)$.

posted per job seeker, as in the standard model, but also by adjusting the skill mix of vacancies towards the relatively more profitable type, which is high-skill vacancies in the model. Given that high-skill workers qualify for both types of jobs, and search on the job is manageable, high-skill employability is less vulnerable to recessions. On the contrary, low-skill workers who qualify only for low-skill jobs are subject to unfavorable shifts in the vacancy mix in recessions.

4 Analysis of the Dynamic Model

I now proceed with characterizing the dynamic version of the model outlined in section 2. I first describe the calibration of the model, and I subsequently simulate the model and describe the dynamic evolution of key variables: high- and low-skill exit rates from unemployment, job-to-job transition rate, over-qualification rate, and high- and low-skill unemployment rates. The calibration of the model is summarized in Table 1.

4.1 Calibration

I consider the high-skill type as representing workers who hold at least a college degree. I therefore set the proportion of high-skill workers to $\delta = 0.25$, which based on the March CPS Annual Demographic Survey Files for the period from 1964 to 2003, equals the average proportion of U.S. labor force that holds a college degree or more. I choose the model period to be one quarter and therefore set the discount rate to r=0.012. For the matching function I make the standard choices. I assume a Cobb Douglas functional form so that $m = z^a v^{1-a}$, and choose an elasticity parameter $a = 0.4$, which lies at the lower range of estimates reported in Petrongolo and Pissarides (2001). I also make a standard choice for the worker's bargaining power. I assume that workers and firms split the surplus equally, i.e., $\gamma = 0.5$.

Following the literature, I select values for the separation rates, s_h and s_l, which are higher than the empirical measures of transition rates from employment to unemployment, to take into account workers who exit the labor force, but whose behavior is similar to those counted as unemployed.[9] Blanchard and Diamond (1990) show that in the U.S., the

[9] Since Clark and Summers (1979) it became eminent that the distinction between the pool of unemployed and the pools of those out of the labor force is fuzzy, with many workers going back an forth between the two states.

"want-a-job" pool in the stock of those not in the labor force is roughly equal to the stock of unemployed. Moreover, they document that only half of the average flow into employment comes from unemployment, with the other half coming from people classified as not in the labor force, signifying that "out of the labor force" job seekers also take part in matching. Assuming that all people classified as out of the labor force participate in the matching process sets an upper bound to the value of the separation rate, which can be computed by adding together the flows from employment to unemployment and out of the labor force. A lower bound can be computed by looking only at flows to unemployment, assuming that only those classified as unemployed search for jobs. To calculate these upper and lower bounds, I use the monthly estimates of transition rates from employment to unemployment and out of the labor force, for college and non-college graduates, reported in Nagypal (2004). After converting the monthly estimates into quarterly frequencies, I find that s_h should lie in the range [0.013-0.041] and s_l in the range [0.032-0.077].[10] I chose to set $s_h = 0.03$ and $s_l = 0.07$, which puts more weight on low-skill separations, and results in an average separation rate in the model of 0.06, which is line with CPS estimates of Hall (2005), when roughly half of the flows from employment to out of the labor force are flows into a job seeking state. Note that the calibrated separation rates do not account for job-to-job transitions. For high-skill workers job-to-job transitions in the form of upgrading to higher job levels, are endogenous in the model. For low-skill workers they are not. But since the focus of the analysis is unemployment differences across skill-groups, I choose not to include job-to-job flows, because workers who directly move into a new job are not accounted as unemployed.

For the parameter values for job creation costs I construct an upper bound as follows. According to Hamermesh (1993), in 1990 average recruitment and training costs in the U.S. represent about on-sixth of average annual labor earnings. Moreover, the job creation costs cannot be too large relative to aggregate output in the model. The standard upper bound in the literature is 5% of output devoted in job creation activities. Based on these

[10]As far as I know, estimates for the U.S. of flows from employment to unemployment and out of the labor force for different educational groups, can only be found in Fallick and Fleischman (2001) who uses the basic monthly CPS survey from February 1994 to December (2000), and Nagypal (2004) who expands the period to January 2004. Using average employment shares by education, from the March CPS Annual Demographic Survey files, and Nagypal's estimates, I first calculated the monthly separation rates for college graduates and workers without a college degree. Then, by counting paths in a probability tree, I derived the quarterly rates as: $s_m[(1-f_m)^2 + f_m s_m] + (1-s_m)[s_m(1-s_m) + s_m(1-f_m)]$, were s_m is the monthly separation rate and f_m the monthly job finding probability. I am grateful to Bruce Fallick for providing me with the CPS estimates of monthly job finding rates by education.

two observations, I set $c_l = 0.13$ and $c_h = 0.22$, which are roughly equal to one third of quarterly low- and high-skill wages, respectively, when the latter are suitably matched with high-skill jobs. With these values, the simulated average vacancy costs in the model are less than 5% of simulated output.

I next turn to the calibration of high- and low-skill productivities, α_{hh} and α_{ll}, the productivity of over-qualified high-skill workers, α_{hl}, and the opportunity costs of working, b_h and b_l. These parameters are selected to match statistics from the simulated data to empirical measures of, i) wage differences between college educated and non-college educated workers, ii) wages differences between over-educated and correctly matched workers, ii) average job finding rate, and iv) unemployment rates of workers with college and less than college education. To match these statistics, I set $\alpha_{ll} = 0.4$, $\alpha_{hh} = 0.68$, and $\alpha_{hl} = 0.6$. The values for the opportunity costs of working are set to $b_h = 0.52$, and $b_l = 0.28$, which are less than the simulated average high- and low-skill wages, respectively. Below I discuss my choice of the relevant targets.

I begin with my choice of target for the wage difference between workers with college and less than college education. Based on the March CPS, Autor et al. (2008) find that the college-plus to high school log wage premium (i.e., the average log wage ratio of college to high school graduates) ranges form 0.4 to 0.65 in the period from 1963 to 2005. This implies an average log wage premium of approximately 0.5.[11] The low-skill group in the model is not restricted to high-school graduates only; it also includes workers with some college education and workers with less than high-school education. However, the average share of employed workers who have not completed college education, but have some college education is not more than 0.25. Therefore, I consider an average log wage premium of 0.5 as a fair target.

My choice of parameter value for the productivity of over-educated college graduates was guided by evidence on wage differentials between over-educated and correctly matched workers. For the U.S., Sicherman (1991) finds that over-educated workers earn more than their co-workers who are not over-educated, but less than similar workers with the same level of schooling that work in jobs that require their actual level of schooling (i.e., correctly allocated workers). In particular, the wage rate of over-educated workers is on average 5% lower than that of correctly allocated workers. Considering this as a lower bound, I choose the value of α_{hl} that implies that the wage of an over-educated college graduate is 10% lower than the wage of a suitably matched college graduate.

[11] Estimates in the same range can also be found in Wheeler (2005).

The average job finding rate I choose to target, incorporates out of the labor force job seekers, in line with my choice of separation rates. I make use of the Hall (2005) estimate that incorporates this group into the group unemployed. Hall took advantage of the expanded unemployment rate series, available from the BLS starting in 1994, which includes those classified as discourage workers who want a job but believe a job is unavailable for several reasons, and those marginally attached to the labor force, who indicate a likelihood of returning to the labor force in the near future. The series was approximated for earlier years by regressing the expanded series to the standard unemployment rate for the years 1994 through 2004 and using the fitted value for the years before. After extending the expanded unemployment rate series to earlier years, the job finding rate was calculated as the ratio of new hires to the number of job seekers, as measured by the expanded unemployment rate series. For the period from 1964 to 2003 the estimated monthly job finding rate averages to 0.28, which works out to an average quarterly job finding rate of about 0.6.

Consistent with my choice of separation and job finding rates, the targeted unemployment rates are higher than the official empirical measures, to take into account workers classified as out of the labor force who participate in the matching process. Unfortunately, the series of marginally attached or discouraged workers in the BLS is not available by education. Therefore, the methodology of Hall, of imputing the expanded unemployment rate series for earlier years using the years after, cannot implemented to construct expanded unemployment rate series by education. Instead, guided by the Blanchard and Diamond (1990) finding that the want-a-job group is roughly equal to the number of unemployed, I approximate the expanded unemployment rates as $\frac{2u_t}{u_t+l_t}$, were u_t is the number of unemployed and l_t is the size of the labor force. Based on the March CPS Annual Demographic Survey files from 1964 to 2003, this calculation yields an average unemployment rate of 0.044 for college graduates, and 0.114 for workers with less college education. The resulting average unemployment rate of 0.10 in the model is consistent with the average expanded unemployment rate in Hall (2005).

Finally, I turn to the calibration of the aggregate productivity process. I approximate through a 9-state Markov chain the quarterly deviations from a linear trend of the U.S. GDP for the period from 1964 to 2003. The estimated autocorrelation coefficient of the standard AR(1) model is 0.9139 and the standard error of the innovation is 0.0084. Hall (2005) and Shimer (2005b) show that the standard model, along the lines of Mortensen and Pissarides (1994), can explain the magnitude of cyclical changes in unemployment only by assuming implausibly large productivity shocks. The reason is that for reasonable calibrations, the magnitude of fluctuations in the vacancy-unemployment ratio is small relative to the fluctuations in the data. The present model, which incorporates on-the-job search and two-sided skill heterogeneity performs considerably better in this dimension.[12] Still, as it will be discovered below, it somewhat underpredicts the volatility of unemployment. However, the results discussed below are not sensitive to this caveat of the mode, because the focus of this paper is on the relative responses of high- and low-skill unemployment rates to the same underlying shock process. Moreover, when the magnitude of fluctuations in the vacancy-job seekers ratio is higher, the magnitude of employment fluctuations is higher, but the qualitative implications are still the same.

Before I proceed with describing the results of the simulations, note that with regard to surplus differences across jobs, the calibrated dynamic model is similar to the steady-state model in the previous section. With the calibrated productivity values, high-skill workers are more productive than low-skill workers, not only when employed in high-skill jobs, but when employed in low-skill jobs as well. However, given that the calibrated value of b_h is higher than the value of b_l, the net productivity of over-qualified workers is lower than the net productivity of low-skill workers. Hence, in the simulations that follow, although over-qualified receive higher wages, they generate lower surplus than suitably matched low-skill workers, despite the much higher exogenous separation rate of the latter. Therefore, when high- as opposed to low-skill workers occupy low-skill jobs the average surplus of low-skill jobs falls. In addition, the surplus of high-skill jobs, is by far higher than the surplus of low-skill jobs, which as mentioned earlier, creates incentives for over-qualified workers to search on the job.

4.2 Simulations

With all the parameter values assigned, I use the free entry conditions given by equations (17) and (18) to find the state-contingent market tightness θ_t and fraction of low-skill vacancies η_t. I then simulate the model as follows: first, I generate a sequence of random aggregate state realizations; then, starting with the first realization of aggregate state, and an initial distribution of employment $e_t = \left\{e_t^{hh}, e_t^{hl}, e_t^{ll}\right\}$, I use the flow equations in (1) to compute the new distribution of employment at the beginning of the next period; and then I repeat. At the end of each period, I record the values of the variables of interest along the sequence of aggregate state realizations.

[12]On-the-job search as an amplification mechanism has been emphasized in Krause and Lubik (2007), while the role of worker heterogeneity in the propagation of shocks has been emphasized in Pries (2007)

Based on the above calibration, on average 84% of vacancies are low-skill. Therefore, job seekers meet low-skill vacancies more frequently; with 0.59 being the average rate by which workers meet low-skill vacancies and 0.11 the average rate by which workers meet high-skill vacancies. The average matching rate is 0.87 for low-skill vacancies and 0.49 for high-skill vacancies, resulting in average matching rate for firms of 0.81. It follows that the significantly lower unemployment rate of low-skill workers can be sustained by some high-skill workers resorting to temporary employment in low-skill jobs. The simulation yields that on average 17.8% of college graduates are over-educated. This figure is well in line with the available empirical measures, of 17 to 18 percent, reported in the introduction.[13]

The resulting average job-to-job flows as a share of employment and total separations, respectively, are 0.02 and 0.4. These measures are comparable, but still lower than the corresponding monthly estimates for college graduates, reported in Nagypal (2004), of approximately 0.02, and 0.55, respectively. This is not is not puzzling; on the contrary, this is what one should expect given that the model captures only transitions to higher job levels (i.e., upgrading to jobs with higher skill requirements), and overlooks transitions to jobs of the same level, while such a distinction is not done in the data. Moreover, some caution may be mandated since the data in Nagypal cover only the period from February 1994 to January 2004 in which the U.S. economy has experienced one expansion and only a mild recession. A longer series would cover additional recessions, and the severe contraction at the beginning of the 80s. Therefore, it would probably yield lower averages. This leads me to conclude that the job-to-job quit rate in the model is reasonable, given that it captures only upward transitions, and corresponds to the longer period from 1964 to 2003.

I now turn to the cyclical behavior of the simulated series. To illustrate how accurately the cyclical behavior of the unemployment rates in the model matches the data, I simulate the model along a series of aggregate productivity realizations that replicates the U.S. GDP log deviations from trend for the period from 1964 to 2003. As shown in Figure 4, with 9 productivity states the replicated series matches quite well the empirical series. I then

[13]Unfortunately, other empirical over-education measures for the U.S. are hard to find, especially referring to college graduates alone. To measure mismatch rates Barlevy (2002) uses a question in the PSID asking employed workers whether they have been thinking of getting a new job. He finds that the fraction of employed workers thinking of getting a new job ranges from 9.6% in 1967 to 17.8% in 1984. In the model, on average 18.6% of college graduates are over-educated. I find this measure to be well in line with Barlevy's estimates. As I also argue at the introduction, given that they can perform a wider range of job types, skilled workers are more likely to be employed as opposed to unemployed job seekers. Therefore, it is reasonable to expect their mismatch rates to be above the average.

compare the simulated unemployment rates to the empirical unemployment rates for the same period.

The empirical series traced in Figure 5, refer to the expanded yearly unemployment rates for college educated and non-college educated workers, constructed as described above. Obviously, the unemployment rate of workers with less than college education fluctuates more than the unemployment rate of college graduates. For the former, the standard deviation of the differences between it and a yearly trend equals 0.021 with differences from trend ranging from -0.307 to 0.059; the standard deviation of the differences between the unemployment rate of college graduates and a yearly trend is 0.0086, with differences from trend ranging from -0.0126 to 0.0187. Hence, the unemployment rate of workers with less than college education is 2.5 times more volatile than the unemployment rate of college graduates, as measured by the ratio of the standard deviations of differences from trend.

The simulated quarterly high- and low-skill unemployment rates are reported in Figure 6. In line with the data, both unemployment rates are strongly countercyclical, but the low-skill unemployment exhibits much more volatility. Figure 7 compares the deviations from a yearly trend of the empirical unemployment rates after averaging over quarters, to the deviations from a yearly trend of the simulated unemployment rates. Clearly, the magnitude of the simulated deviations is smaller. As mentioned earlier, this corresponds to the general failure of the matching model to match the empirical volatilities. But, in terms of relative volatilities the model performs quite well. The standard deviation of the differences between the simulated low-skill unemployment rate and a trend is 2.6 times higher than the corresponding standard deviation for the simulated high-skill unemployment rate, compared to 2.5 times in the data.

The behavior of exit rates from unemployment is also traced in Figure 6. As expected, the matching rate $m(\theta)$, which corresponds to the rate at which high-skill workers exit unemployment, is procyclical. The low-skill exit rate from unemployment depends also on the skill mix of vacancies. The insights from the analytic exercise above carry over to the dynamic version of the model. In downturns firms open fewer vacancies of both types, but relatively fewer low-skill vacancies. Therefore, the skill mix of vacancies moves countercyclically; firms upgrade the skill mix of vacancies in recessions and downgrade it in booms. Although not obvious to the naked eye, the low-skill exit rate from unemployment exhibits relatively higher volatility, reflecting the countercyclical behavior of the vacancy mix. The standard deviation of log deviations from a quarterly trend is 0.0370 for the low-skill, and 0.0331 for the high-skill exit rate.

The model is also consistent with the salient regularity that job-to-job transition rates are procyclical, and evidence that the quality of job-worker matches is lower in recessions. As shown in Figure 6, despite the countercyclical behavior of the skill mix of vacancies, which implies that workers are relatively more likely to meet high- as opposed to low-skill vacancies in recessions, both of these features are present in this model. A higher fraction of high-skill workers relocates into high-skill jobs in booms, while the over-qualification rate (i.e., the fraction of over-qualified high-skill workers) moves countercyclically, with a lag of two quarters relative to high-skill unemployment. The lag is not surprising, since it takes time for unemployed workers to arrive to jobs.

Overall, the model captures important features of the data. The section that follows elaborates upon the underlying mechanisms that drive the results.

4.3 Responses to Aggregate Productivity Shocks

This section goes deeper into the mechanisms that lie beneath the dynamic responses of variables to aggregate productivity shocks. I demonstrate the consequences of a fall in aggregate productivity from $y = 1.0346$ to $y = 1$. This switch represents a reduction of approximately 1.5 standard deviation in output. To illustrate the various effects I simulate the model as calibrated in section 4.1 with $y = 1.0346$ until the endogenous variables converge to a stable value. I then set $y = 1$ and simulate the effects, until the endogenous variables converge to a new stable value.[14] The results of this exercise are summarized in Figure 8.

I begin with the conventional result that in recessions firms open fewer vacancies per job seeker. As soon as the negative shock arrives, the number of vacancies and meeting rate decline. The number of vacancies rises afterwards as rising arrival rates of job seekers to firms, encourage more job openings, but never reaches its initial level. The exit rates from unemployment follow a similar pattern; they decline on impact, and subsequently recover partial of the initial decline, reflecting the moderate increase in the number of vacancies.

On impact, the composition of job seekers shifts towards more unemployed and fewer over-qualified ($\psi_t\varphi_t$ and $\psi_t(1-\varphi_t)$ increase), reflecting the drop in exit rates from unemployment. Moreover, the fraction of high-skill job seekers $(1-\psi_t\varphi_t)$ declines. This shift in the composition of job seekers entails a larger effective pool of jobs seekers for firms with low-skill vacancies, and a smaller one for firms with high-skill vacancies. Still, it is not

[14]Despite the assumed sample path, the value functions used in the simulations assume that aggregate productivity obeys the calibrated AR(1) process as described earlier.

sufficient to encourage firms to downgrade the skill mix of vacancies. The share of low-skill vacancies declines on impact, reflecting the productivity "scale" effect, which as emphasized in section 3, makes high-skill jobs relatively more profitable in recessions. Despite the subsequent increase in vacancies, the fraction continues to decline, because the upturn in the number of high-skill vacancies is higher than the upturn in the number of low-skill vacancies. This is because of the subsequent increase in the fraction of over-qualified (or equivalently, decline in the fraction of unemployed) job seekers, which reinforces the scale effect, by making high- as opposed to low-skill vacancies even more attractive to firms. As firms post relatively more high-skill vacancies, at the onset of the recession, the fraction of over-qualified high-skill workers declines slightly. However, it rises afterwards as a higher number of unemployed high-skill workers arrives to firms with low-skill vacancies, and converges to a higher level.

It follows that the burden of recessions falls more heavily on low-skill workers, both because the relative profitability of low-skill vacancies is lower in recessions, but also because the number of over-qualified job seekers, who congest the low-skill market, increases in recessions. Over-qualified workers lower the chances low-skill workers find jobs both directly, by making it more difficult for low-skill workers to locate jobs, and indirectly by lowering the profits of low-skill jobs, and discouraging firms from opening low-skill vacancies. As mentioned earlier, when low-skill jobs are occupied by over-qualified instead of suitable workers, their surplus declines on average, because the former quit sooner. However, one has to keep in mind that in the presence of matching imperfections (i.e., given that workers cannot ex-ante target jobs) congestion at the lower segment of the labor market is inevitable in recessions even when high-skill workers refuse the low-skill jobs they encounter. In this case, instead of over-qualified job seekers, a higher number of unemployed high-skill job seekers who refuse to match, congests the low-skill segment. Hence, it not clear-cut that low-skill employability improves when high-skill workers refuse low-skill jobs. The task of clarifying the consequences of transitory over-qualification on low-skill employability is taken up in the following section.

4.4 The Consequences of Cross-Skill Matching

The question I ask in this section is whether low-skill workers are better off when high-skill workers reject the low-skill jobs they encounter, instead of accepting them transitorily. The answer to this question depends on how transitory over-education affects the profits of low-skill jobs, and thus the number of low-skill vacancies opened per job seeker.

When high-skill workers accept low-skill job offers, they affect the profits of low-skill jobs in two ways. First, by raising the effective matching rate of firms with low-skill vacancies, they raise their profits. Firms with low-skill vacancies are better off hiring low- instead of high-skill workers since the latter are more likely to quit, and thus generate lower surplus. Nevertheless, they are still better off when an over-qualified worker fills the vacancy instead of the vacancy remaining unfilled. This positive impact can also be interpreted as lower recruitment costs for low-skill firms, because they can fill their vacancies faster, and is captured by the second term in the free-entry condition (17), which would be absent if high-skill workers refused to match with the low-skill jobs they met.

The second consequence on the profits of low-skill jobs is negative, and arises when high-skill workers crowd out low-skill workers from low-skill jobs. Crowding out occurs when the number of over-qualified high-skill workers increases at the cost of a lower number of correctly matched low-skill workers, who instead remain unemployed. In particular, over-qualified workers push low-skill workers into unemployment by congesting the low-skill market, thus making it harder for firms with low-skill vacancies to locate low-skill workers who are better suited for the jobs. When this occurs, the profits of low-skill vacancies may decline on average, as the decline in the "quality" of low-skill matches may outweigh the positive impact of a higher effective matching rate. Looking at the free-entry condition for low-skill vacancies, the crowding out effect translates into a reduction in $\psi_t \phi_t$ due to a rise in the number of overqualified workers e_t^{hl}. Naturally, when high-skill workers accept low-skill jobs u_t^h is lower. Hence, even if u_t^l is higher, $\psi_t \phi_t$ may decline if e_t^{hl} is sufficiently high.

On the other hand, when high-skill workers refuse the low-skill jobs they encounter, firms with low-skill vacancies do not suffer the negative quality effect arising from over-qualified workers crowding out low-skill ones. But in the meantime, they do not benefit from the arrival of unemployed high-skill workers either. Instead, the high-skill unemployed congest the low-skill market, making it more difficult for low-skill firms to fill vacancies. Consequently, if the crowding out effect is smaller than the positive impact of higher effective matching rates for firms with low-skill vacancies, low-skill workers are actually better off when high-skill workers accept the low-skill jobs they encounter.

The question that follows is whether the positive effect of a higher effective matching rate dominates the crowding out effect. To answer this question I simulate the calibrated model assuming that when high-skill workers arrive at low-skill firms they refuse to match, and compare the results to the case cross-skill matching occurs, as above. To isolate the impact of cross-skill matching on the lower segment of the labor market, the only change I bring into the model without cross-skill matching is that when high-skill workers arrive at low-skill jobs a match is not formulated. Plugging $e_t^{hl} = 0$ into the free-entry conditions (17) and (18), and setting the second term in (17) equal to zero bring us to the free-entry conditions for the model without cross-skill matching. I assume that firms do not internalize the fact that high-skill workers refuse the low-skill jobs they meet. Hence, the surplus expressions remain as in (14) to (16). The underlying assumption is that high-skill workers refuse low-skill jobs because of some idiosyncratic reasons, which are unknown to the employers. Such reasons could be for instance high on-the-job search costs, or high levels of disutility due to the worker's dissatisfaction for being underemployed.[15] The results of this comparison are summarized in Figure 9, where I trace the responses of the model with and the model without cross-skill matching to the same negative shock as in the previous section.

Surprisingly enough, the comparison reveals that by accepting transitorily low-skill jobs, high-skill workers improve the employment prospects of low-skill workers. The low-skill exit rate from unemployment is higher, and the low-skill unemployment rate is lower in the model with cross-skill matching, suggesting the crowding out effect is relatively small. Indeed, with the presence of on-the-job searchers, high-skill workers capture a higher share in the pool of job seekers (i.e., $\psi_t \phi_t$ is lower), despite the fact that high-skill unemployment is lower. Consequently, high-skill workers arrive to firms relatively more frequently. For this reason, firms post relatively more high-skill vacancies when cross-skill matching occurs. But still, the exit rate from unemployment of low-skill workers is higher, because the meeting rate is higher. That is, there are more vacancies available per job seeker.

As it turns out, when high-skill workers occupy transitorily low-skill jobs, firms open more vacancies of both types, because the pools of potential hires are larger at both segments of the labor market. Firms open more low-skill vacancies, because the benefit of a higher effective matching rate outweighs the negative quality effect of crowding out. But firms open more high-skill vacancies as well, because as mentioned above, high-skill workers capture a higher share in the pool of job seekers. Hence, firms with high-skill vacancies also benefit

[15] I make this assumption both because I think it is reasonable, and because it is convenient. I think it is reasonable, because, irrespective of whether the worker will accept or not a low-skill job offer once such an opportunity arises, a low-skill job is always an option for the high-skill worker, something that the employer needs to take into account. It is convenient, because if firms internalized that high-skill worker accept only high-skill jobs, high-skill jobs would generate a higher surplus. Isolating the impact of cross-skill matching alone would be more difficult in this case.

from a higher effective matching rate. Overall, the willingness of high-skill workers to accept low-skill jobs, and the resulting higher search activity at both segments of the labor market, maintains a higher incentive for firms to open vacancies, thus facilitating exit from unemployment. Therefore, both the rate by which workers meet high-skill vacancies and the rate by which workers meet low-skill vacancies are higher. It follows that in the model with cross-skill matching, in addition to the higher number of employed low-skill workers, the number of employed high-skill workers is higher, not only because of the over-qualified job seekers, but also because the number of correctly allocated high-skill workers is higher, as shown in Figure 10.

I now turn to the consequences of cross-skill matching on the cyclical responsiveness of high- and low-skill unemployment rates. In Figure 11, I compare the percentage deviation responses to the same shock as above. I find that when high-skill workers accept low-skill jobs, firms respond to negative productivity shocks by lowering the number of vacancies opened more drastically, while keeping the skill-mix of vacancies relatively more stable. As shown in the figure, with cross-skill matching the percentage decline in the fraction of low-skill vacancies is lower, while the percentage decline in the number of vacancies posted is higher in both sectors. This in turn, translates into a higher percentage decline in the meeting rate and a higher percentage decline in the exit rates from unemployment for both types of workers. As firms react by reducing job openings more drastically, but more evenly across the two sectors, as opposed to concentrating reductions at the low-skill sector, in relative terms, cross-skill matching moderates the impact of recessions on low-skill unemployment, but amplifies the responsiveness of unemployment overall. As can be verified in the figure, both unemployment rates rise more eminently in response to the negative shock in the model with cross-skill matching, but the high-skill unemployment rate rises by much more.

The main reason for this becomes apparent when considering the evolution in the skill composition of job seekers. In recessions the skill composition of job seekers favors firms with low-skill vacancies and hurts firms with high-skill vacancies, irrespective of whether high-skill workers accept low-skill jobs or not. This is because the majority of labor force is low-skill, therefore low-skill workers are over-represented in the pool of unemployed. With cross-skill matching, this effect is reinforced. In periods of rising unemployment firms with low-skill vacancies benefit from higher arrival rates not only of low-skill unemployed, but also of high-skill unemployed who would otherwise only cause congestion, and vice versa for firms with high-skill vacancies; on-the-job searchers who expand the pool of potential hires for firms with high-skill vacancies, capture a higher share in the pool of job seekers in booms, when there are fewer unemployed job seekers. In turn, this moderates the negative impact of recessions on the relative profitability of low-skill vacancies, and limits the scope of concentrating reductions in job openings at the low-skill sector.

5 Directed Search

A question that arises is whether eliminating matching imperfections, so that job seekers arrive only at the jobs they are best suited for, would yield lower unemployment rates for college graduates, simply because there are larger productivity gains associated with skilled positions, and thus, the market for college graduates is tighter. To investigate this possibility, I consider the case workers can distinguish the type of vacancy before they apply, and therefore, can target only the jobs they are best suited for (i.e., search is directed). In this case, cross-skill matching is not possible, the two sub-markets are separated, and matching in one sub-market is independent of the conditions in the other market. Hence, the number of vacancies per job seeker in each sub-market depends only on the surplus jobs generate in each sub-market. Consequently unemployment rate differences are driven mainly by differences in the surpluses of the two types of jobs.

In the model with directed search a worker of type i, locates a vacancy of the same type at rate $m(\theta_t^i)$, where $\theta_t^i = \frac{\nu_t^i}{u_t^i}$, and firms locate suitable workers at rate $q(\theta_t^i)$. Given that cross-skill matching does not occur the free-entry conditions in each market take the following form

$$(1-\gamma)\beta E_t S_{t+1}^{ll} = \frac{c_l}{q(\theta_t^l)} \tag{26}$$

$$(1-\gamma)\beta E_t S_{t+1}^{hh} = \frac{c_h}{q(\theta_t^h)} \tag{27}$$

and the surplus functions are given by

$$S_t^{ll} = y\alpha_{ll} - b_l + \beta E_t[(1-s_l)S_{t+1}^{ll} - \gamma m(\theta_t^l)S_{t+1}^{ll}] \tag{28}$$

$$S_t^{hh} = y\alpha_{hh} - b_h + \beta E_t[(1-s_h)S_{t+1}^{hh} - \gamma m(\theta_t^h)S_{t+1}^{hh}] \tag{29}$$

Notice that the composition of job seekers is no longer relevant in determining the number of vacancies posted of each type. In deciding how many vacancies of each type to open, firms consider only the recruitment costs associated with each type (right-hand-sides of (26) and (27)), and the surplus each job generates. In turn, the surplus depends on workers' net productivity and tightness in each sub-market, as reflected in the meeting rates $m(\theta_t^l)$ and $m(\theta_t^h)$.

Given this set up, the question I ask is whether productivity differences between college graduates and workers with less than college education, as reflected in the wages they earn, is what is hidden behind the higher exit rates from unemployment of high-skill workers. To address this question I simulate the model with directed search, searching for the productivity dispersion that can generate the observed unemployment rate differences between college graduates and workers with less than college education. To simplify things, I focus on the "best-case" scenario for the profitability of high-skill vacancies. In particular, I assume that the cost of filling high-skill vacancies is equal to the cost of filling low-skill vacancies, and that the opportunity cost of employment is the same for both types. The reasonable case is to assume that these values are higher for college graduates, which makes opening vacancies that require college education more costly. Moreover, I keep the lower separation rate for college graduates as calibrated in the previous section, which also makes vacancies with high skill requirements relatively more profitable. By reverse calibration of the unemployment rates, I determine the required net productivity dispersion, while the rest of the parameters are as calibrated in section 4.1.

Despite the parameter choices that favor high-skill vacancy creation, I find that in the absence of matching imperfections, the productivity of college graduates net of the opportunity cost of working, must be more than six times higher than that of lower educated workers, just for the exit rate from unemployment of the former to be higher than that of the latter, as it is empirically observed. In order for the simulated unemployment rates to match their empirical measures, the net productivity of college graduates must be eight times higher than of workers with less than college education. This implies an average log wage premium for college graduates of approximately 0.9, which is highly unrealistic. Unless there are large differences in the matching technology between the two sub-markets or differences in wage setting across the two sectors, this finding suggests that transitory over-education is an important channel through which higher skill groups manage to keep their unemployment rates relatively low, as also suggested by the observed high over-education rates and large job-to-job flows mentioned in the introduction.

6 Conclusion

I have examined the labor market cyclical dynamics in a search and matching model with two-sided heterogeneity and on-the-job search, which accounts for asymmetries in matching technology. More skilled individuals can perform a wider range of job types than less skilled individuals. Matching imperfections imply that sometimes job seekers arrive in jobs they are not qualified to perform, in which case they are not hired. Therefore, more skilled workers are relatively more capable of maintaining high employment rates at all states of the business cycle, given that they are willing to accept jobs below their skill level. The possibility of on-the-job search encourages workers to accept transitorily jobs below their skill level, thereby influencing the employment prospects of lower skill groups. I have calibrated the model to the U.S. labor market, assuming that skilled individuals are college graduates and less skilled individuals are those with less than college education.

My analysis highlights the importance of a cyclical pattern in the matching behavior of skilled workers – of downgrading to lower job levels to escape unemployment, and upgrading by on-the-job search – in explaining why typically unemployment is lower and less cyclical at higher levels of educational attainment. A model without this feature does not predict the observed dramatic unemployment differences between college graduates and workers with less than college education. At the same time, the model is consistent with important features of labor market dynamics such as the highly procyclical rate of job-to-job transitions and evidence that the quality of job-worker matches formed in recessions is lower. I further show that this cyclical pattern in the matching behavior of skilled workers does not harm low-skill employability. In contrast, it maintains incentives for firms to open more of both low- and high-skill vacancies, because this type of behavior expands the pool of potential hires in both sectors.

A nice property of the model is that by allowing for job finding rates to vary endogenously across skill groups, it explains important dimensions of the data, and provides rich insights on the dynamic interaction between skill groups, without introducing complex features relative to the standard search and matching model. The key element of the model is that workers face restrictions in the types of jobs they can perform, conditional on the skills they possess, which is a natural consequence of skill heterogeneity in the labor market. Some other aspects may have important implications. For instance, differences in the matching technology across sectors may also produce differences in labor marker dynamics across skill groups. However, there is little empirical background for such arguments.

Another feature of the model, which has not been explored in detail in this paper, is that it amplifies the responses to shocks. The lack of amplification in the standard model has been emphasized in Hall (2005) and Shimer (2005b). On-the-job search as an amplification mechanism has been emphasized in Krause and Lubik (2007) in a model without worker heterogeneity, while the amplification properties of worker heterogeneity have been emphasized in Pries (2007) in a model without on-the-job search. The present model has both of theses features, and in addition allows for jobs finding rates to differ across skill groups. Synergies between these three features may have some interesting implications for the propagation of shocks.

Finally, in the model job heterogeneity and worker heterogeneity interact in interesting ways; shifts in the composition of jobs seekers, cause changes in the skill composition of new employment opportunities. Consequently, the inclusion of more than two skill types in the model promises important insights on the efficiency of worker reallocation over the business cycle. As already mentioned, the importance of on-the-job search in facilitating the reallocation of workers from bad to better jobs has been previously elaborated in Barlevy (2002). However, in Barlevy such across-skill interactions have been overlooked, by looking only on symmetric equilibria, in which neither the skill composition of job seekers nor the skill composition of vacancies is allowed to change.

Table 1: Calibrated Parameters

Parameter	Value	Description
a	0.4	Match elasticity
r	0.012	Discount rate
γ	0.5	Worker's Nash bargaining share
δ	0.75	P roportion of low-skill individuals in the labor force
α_{ll}	0.4	Productivity of a low-skill worker
α_{hh}	0.68	Productivity of a suitably matched high-skill worker
α_{hl}	0.6	Productivity of an over-qualified worker
b_l	0.28	Low-skill opportunity costs of working
b_h	0.52	High-skill opportunity costs of working
s_l	0.07	Low-skill job destruction rate
s_h	0.03	High-skill job destruction rate
c_l	0.13	Low-skill vacancy costs
c_h	0.22	High-skill vacancy costs

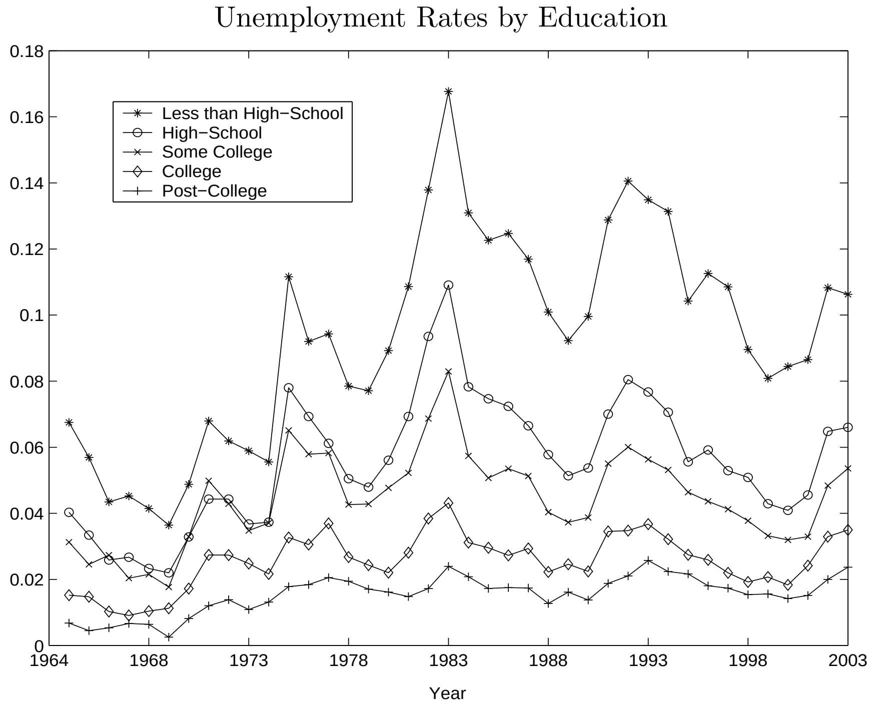

Figure 1: Unemployment rates of civilians between the ages 22 to 65. Source: March CPS, Annual Demographic Survey files.

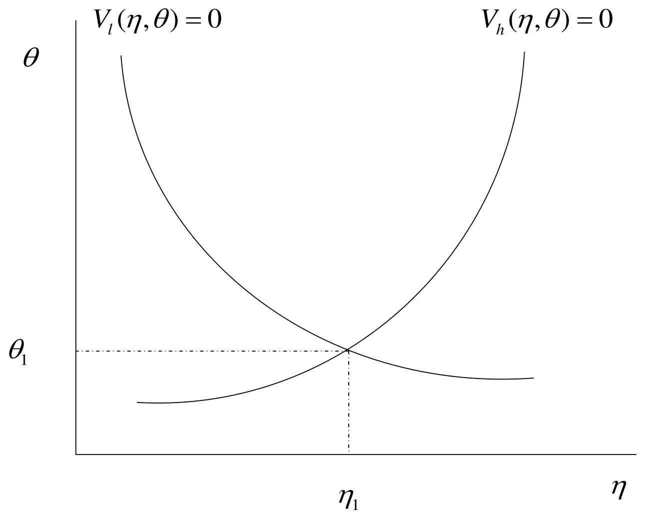

Figure 2: The Steady State Equilibrium

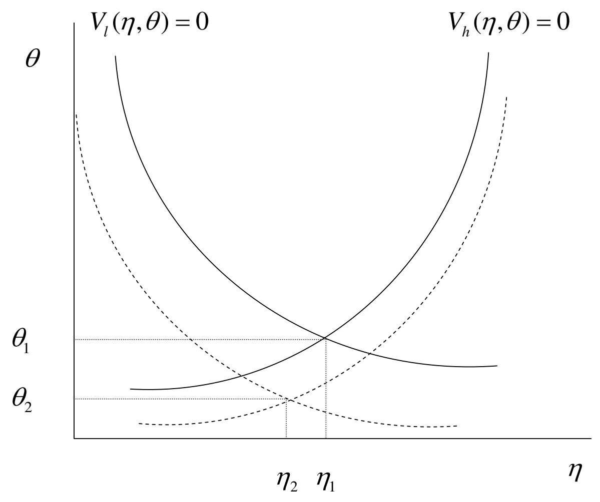

Figure 3: The Impact of a Negative Productivity Shock

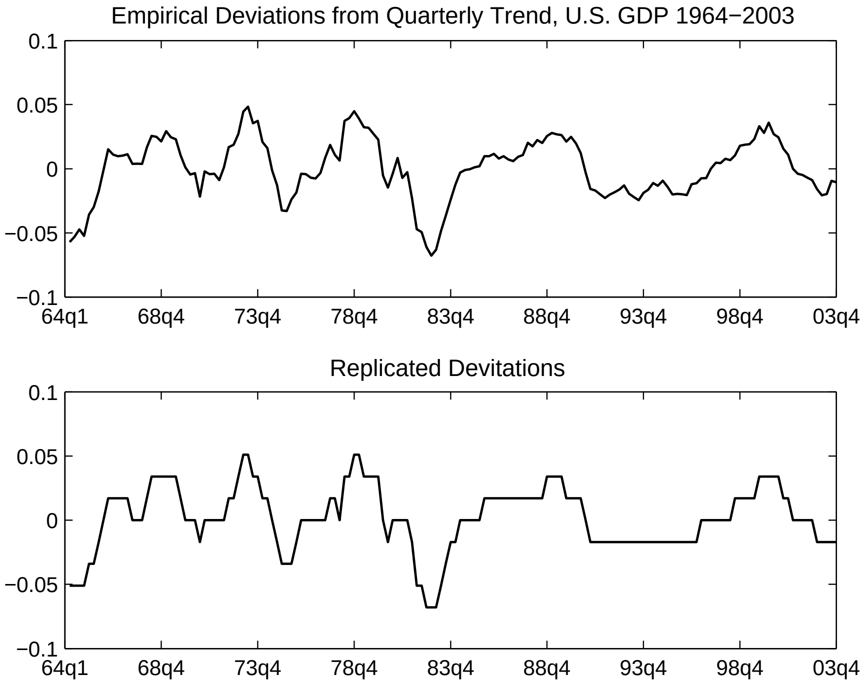

Figure 4: Empirical and Calibrated log Deviations from Trend

Figure 5: Expanded unemployment rates. Source: March CPS Annual Demographic Survey files.

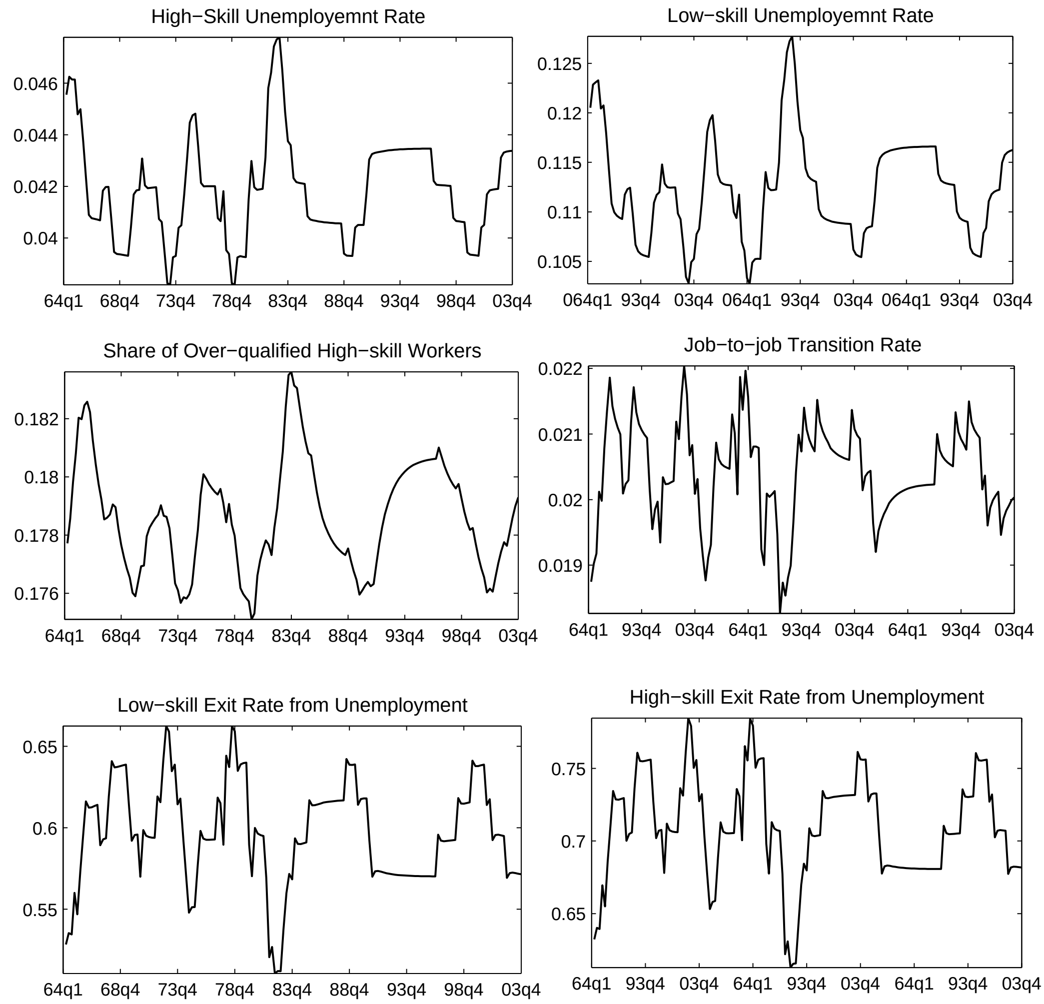

Figure 6: Simulations

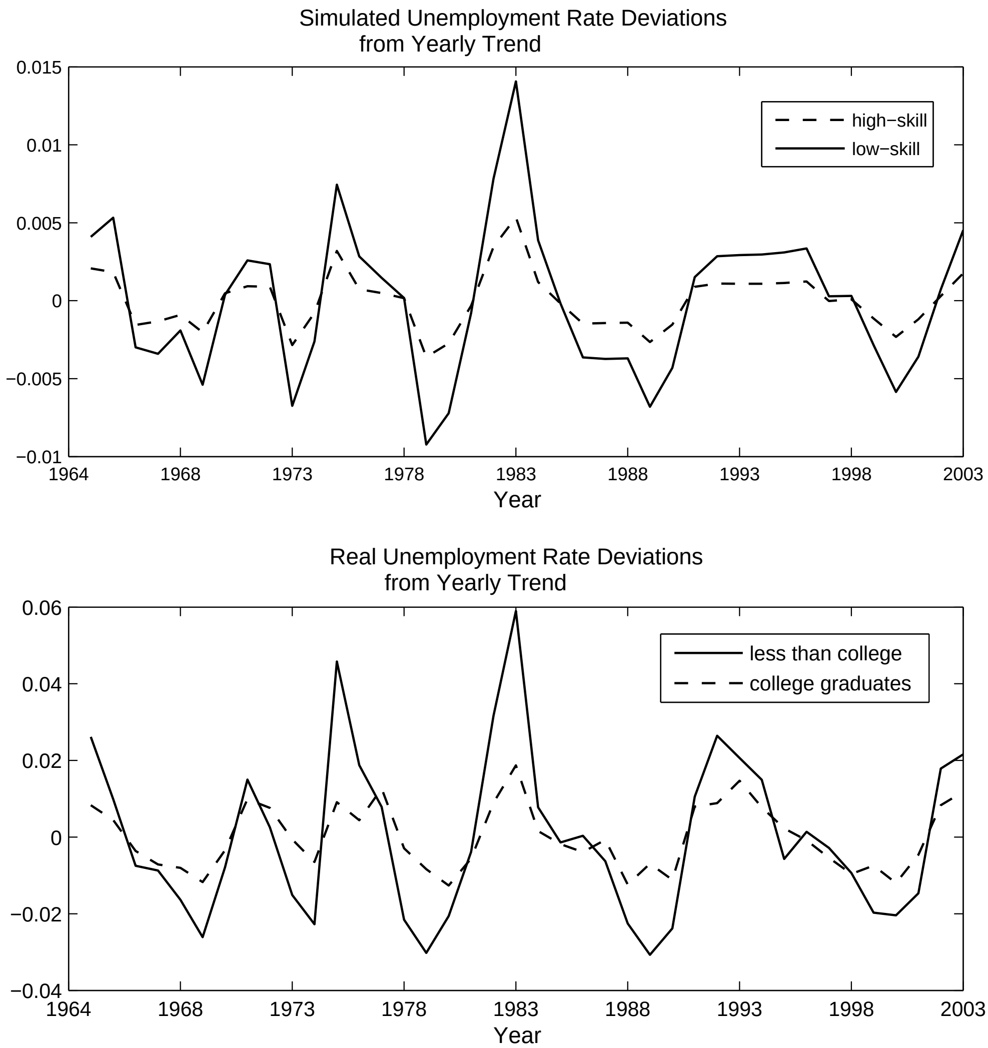

Figure 7: Empirical and Simulated Deviations

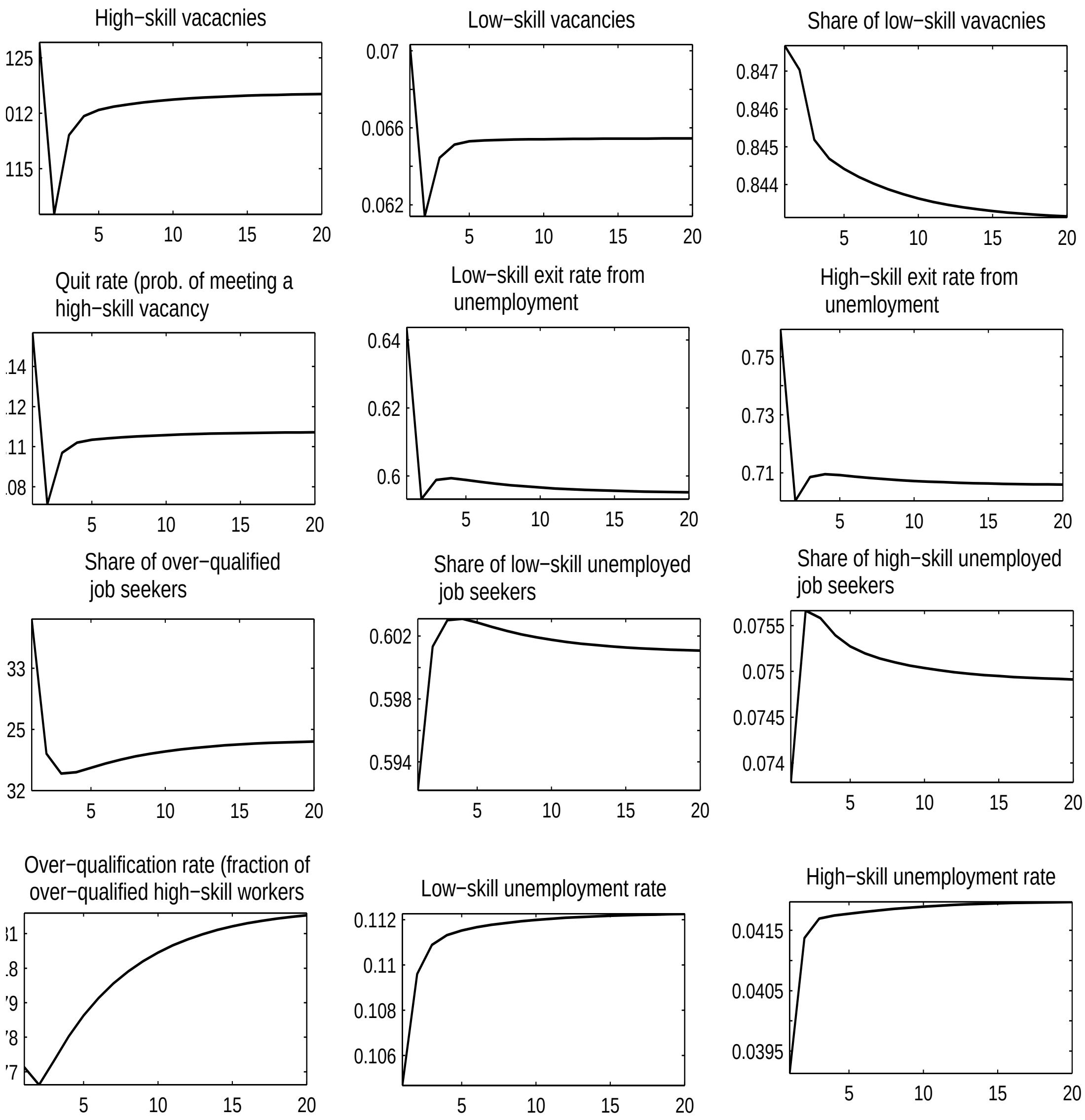

Figure 8: Responses to a Negative Aggregate Productivity Shock

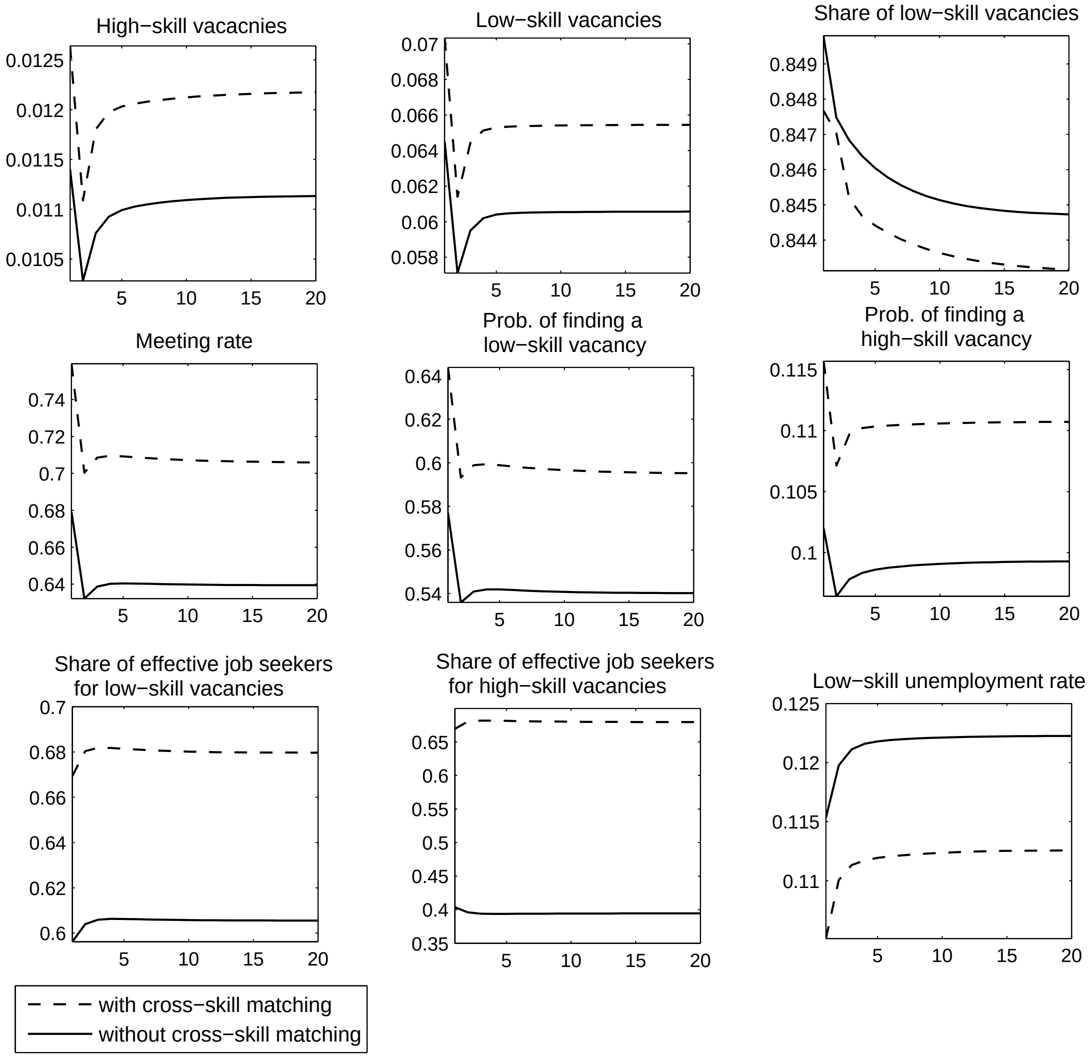

Figure 9: Responses to a Negative Aggregate Productivity Shock

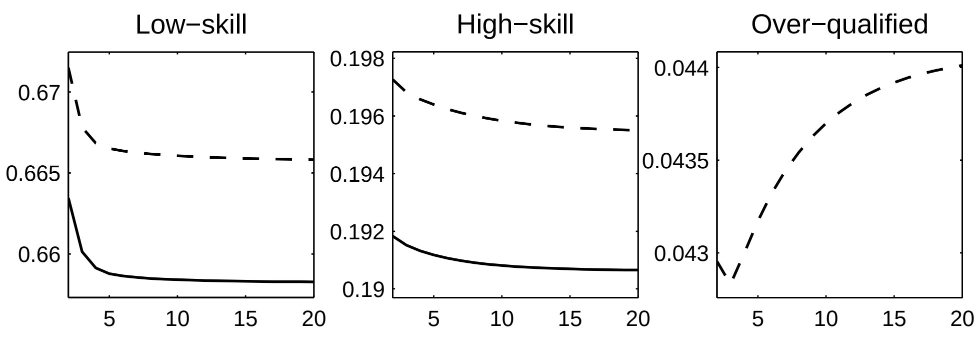

Figure 10: Employment Responses to a Negative Aggregate Productivity Shock

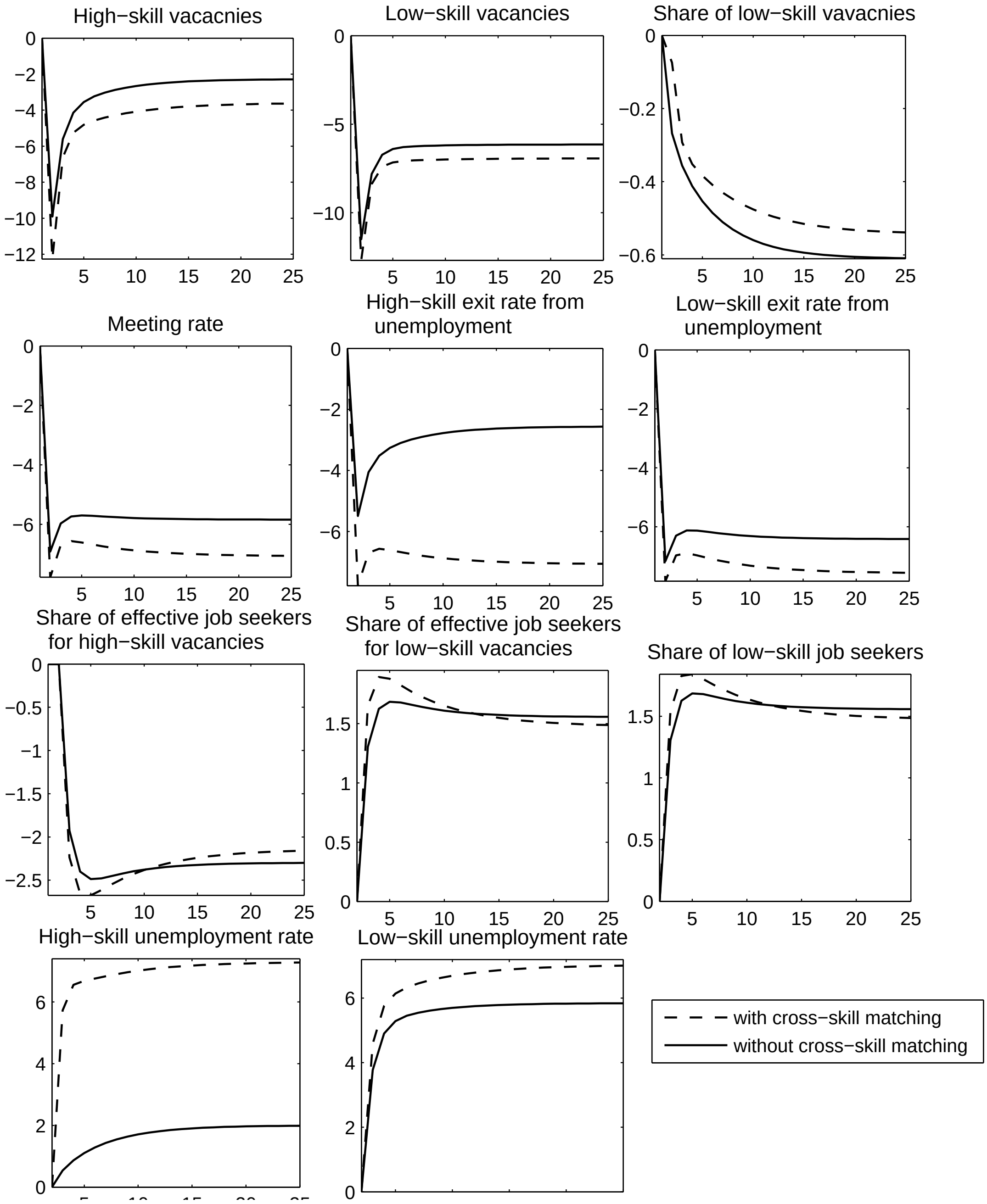

Figure 11: Percentage Deviation Responses to a Negative Aggregate Productivity shock

APPENDIX

The Steady State Equilibrium

Since the steady state distribution $e = \{e^{ll}, e^{hl}, e^{hh}\}$ is constant, by equating the flows in to the flows out of each of the three states, the steady state values of $\psi\varphi$ and $\psi(1-\varphi)$ are uniquely determined for a given value of η and θ as follows

$$\psi\varphi = \frac{\delta(s+(1-\eta)m(\theta))}{\delta(s+(1-\eta)m(\theta))+(1-\delta)(s+\eta m(\theta))} \quad (30)$$

$$\psi(1-\varphi) = \frac{(1-\delta)(s+\eta m(\theta))(s+(1-\eta)m(\theta))}{[\delta(s+(1-\eta)m(\theta))+(1-\delta)(s+\eta m(\theta))](s+m(\theta))} \quad (31)$$

Substituting these expressions together with the surplus expressions (21) to (23) into the free entry conditions (19) and (20) yields a set of equations in terms of the endogenous variables η and θ, which I denote as $F_l(\eta,\theta)$ and $F_h(\eta,\theta)$. To ensure the existence and uniqueness of a steady state equilibrium I define the parameter conditions under which $F_l(\eta,\theta)$ and $F_h(\eta,\theta)$ intersect only once.

First, I specify the conditions under which, for a given η, an increase in θ lowers has a negative effect on both loci. The corresponding partial derivatives with respect to θ are given by

$$\frac{\partial F_l}{\partial\theta} = \frac{\partial(q(\theta)\psi\varphi)}{\partial\theta}S^{ll} + q(\theta)\psi\varphi\frac{\partial S^{ll}}{\partial\theta} + \frac{\partial(q(\theta)\psi(1-\varphi))}{\partial\theta}S^{hl} + q(\theta)\psi(1-\varphi)\frac{\partial S^{hl}}{\partial\theta} \quad (32)$$

$$\frac{\partial F_h}{\partial\theta} = \frac{\partial(q(\theta)(1-\psi\varphi))}{\partial\theta}S^{hh} + q(\theta)(1-\psi\varphi)\frac{\partial S^{hh}}{\partial\theta} \quad (33)$$

I begin with specifying the conditions that ensure $\frac{\partial F_l}{\partial\theta} \leq 0$. The second and last terms in (32) are negative, because $\frac{\partial S^{ll}}{\partial\theta} \leq 0$ and $\frac{\partial S^{hl}}{\partial\theta} \leq 0$. Moreover,

$$\frac{\partial\psi(1-\varphi)}{\partial\theta} = -\frac{\partial m(\theta)}{\partial\theta}\frac{s(1-\delta)[\delta(1-\eta)(s+(1-\eta)m(\theta))^2+(1-\delta)\eta(s+\eta m(\theta))^s]}{(s+m(\theta))^2[\delta(s+(1-\eta)m(\theta))+(1-\delta)(s+\eta m(\theta))]^2} \leq 0 \quad (34)$$

Therefore, given that $q'(\theta) < 0$, the third term is also negative. To complete the proof I need to show that the first term is also negative, which requires $\frac{\partial\psi\varphi}{\partial\theta} \leq 0$. This derivative is given by

$$\frac{\partial\psi\varphi}{\partial\theta} = \frac{\partial m(\theta)}{\partial\theta}\frac{\delta(1-\delta)s(1-2\eta)}{[(1-\delta)(s+\eta m(\theta))+\delta(s+(1-\eta)m(\theta))]^2} \quad (35)$$

which is negative as long as $\eta \geq \frac{1}{2}$. I therefore proceed with specifying the condition that ensures $\eta \geq \frac{1}{2}$ in equilibrium. The ratio of $F_l(\eta,\theta)$ to $F_h(\eta,\theta)$ yields,

$$\frac{(ya_l-b)}{(ya_h-b)}\left[\frac{\delta\lambda_1(s+(1-\eta)m(\theta))}{(1-\delta)\lambda_2(s+\eta m(\theta))} + \frac{\lambda_1(s+(1-\eta)m(\theta))}{\lambda_3(s+m(\theta))} + \frac{\gamma\eta m(\theta)}{\lambda_3}\right] = \frac{c_l}{c_h} \quad (36)$$

which must be satisfied in equilibrium. One can easily verify that the left hand side of this expression declines both with η and θ. Therefore, if when θ goes to infinity and $\eta = \frac{1}{2}$ the left hand side is still greater than $\frac{c_l}{c_h}$, then $\eta \geq \frac{1}{2}$ must hold for the condition in (36) to be satisfied. Evaluating the left hand side at $\eta = \frac{1}{2}$, and taking the limit when $\theta \to \infty$, yields

$$\frac{(ya_l - b)}{(ya_h - b)} \left[\frac{\delta}{(1-\delta)} + \frac{2\gamma}{\gamma+1} \right] \tag{37}$$

If the above expression is greater than $\frac{c_l}{c_h}$, then the first term in (32) is also negative. This completes the set of restrictions that ensure $\frac{\partial F_l}{\partial \theta} \leq 0$.

I now turn to the conditions under which $\frac{\partial F_h}{\partial \theta} \leq 0$. Given that $\frac{\partial S^{hh}}{\partial \theta} \leq 0$, the second term in (33) is negative. To establish that the first term is also negative requires an additional restriction on the matching technology. Namely, that $\frac{\partial q(\theta)(1-\psi\varphi)}{\partial \theta} \leq 0$. This condition imposes a tighter restriction on the elasticity of $q(\theta)$ than what is standardly assumed. The standard assumption (see e.g., Mortensen and Pissarides, 1994) is that the elasticity of $q(\theta)$ with respect to θ is between -1 and 0. As Dolado *et al.* (2004) also argue, compared to the standard matching model, this is the only additional restriction that needs to be imposed on the matching technology. Moreover, numerical simulations show that this derivative is always positive for values of $\delta \geq \frac{1}{2}$.

I next show that $\frac{\partial F_h}{\partial \eta} \geq 0$ whereas $\frac{\partial F_l}{\partial \eta} \leq 0$, so that the two loci have opposite slopes in the $[\theta,\eta]$ plane. Taking the derivative of $F_l(\eta,\theta)$ and $F_h(\eta,\theta)$ with respect to η yields

$$\frac{\partial F_l}{\partial \eta} = q(\theta)\frac{\partial \psi\varphi}{\partial \eta}(S^{ll} - S^{hl}) + q(\theta)\frac{\partial \psi}{\partial \eta}S^{hl} + q(\theta)\psi\varphi\frac{\partial(S^{ll} - S^{hl})}{\partial \eta} + q(\theta)\psi\frac{\partial S^{hl}}{\partial \eta} \tag{38}$$

$$\frac{\partial F_h}{\partial \eta} = q(\theta)\frac{\partial(1-\psi\varphi)}{\partial \eta}S^{hh} + q(\theta)(1-\psi\varphi)\frac{\partial S^{hh}}{\partial \eta} \tag{39}$$

The first three terms in (38) are negative, because

$$\begin{aligned}
\frac{\partial \psi\varphi}{\partial \eta} &= -\frac{\delta(1-\delta)m(\theta)(2s+m(\theta))}{\chi^2} \leq 0 \\
S^{ll} - S^{hl} &= \frac{(1-\eta)m(\theta)}{\lambda_2\lambda_3} \geq 0 \\
\frac{\partial \psi}{\partial \eta} &= -\frac{(1-\delta)m(\theta)}{\chi^2}\left[\begin{array}{c} (s+(1-\eta)m(\theta))[3\delta(1-\eta)+3(1-\delta)\eta+\delta\eta] \\ +\delta(s+\eta m(\theta))\eta m(\theta) \end{array}\right] \leq 0 \\
\frac{\partial(S^{ll} - S^{hl})}{\partial \eta} &= -\frac{(y\alpha_{ll} - b)m(\theta)}{(\lambda_2\lambda_3)^2}\left[\begin{array}{c} \lambda_2(r+s)+\lambda_3(1-\gamma(1-\eta))m(\theta) \\ +\gamma\eta m(\theta)^2 + (1-\gamma)(1-\eta)m(\theta)^2 \end{array}\right] \leq 0
\end{aligned}$$

where $\lambda_1 = [\delta(s+(1-\eta)m(\theta)) + (1-\delta)(s+\eta m(\theta))]$, $\lambda_2 = (r+s+\gamma\eta m(\theta)+(1-\eta)m(\theta))$, and $\lambda_3 = (r+s+\gamma(1-\eta)m(\theta))$. The last term in (38) is positive, because $\frac{\partial S^{hl}}{\partial \eta} \geq 0$. However, adding together the third and last term results in a negative term as long as $\gamma \geq \frac{1}{2}$ and

$\varphi \geq \frac{1}{2}$. Since the exogenous separation rate is the same for both types of workers, whereas the job finding rate of low-skill workers is always lower than that of high-skill workers, it follows that the former are always over-represented in the pool of unemployed. Therefore, $\varphi \geq \delta$ is always satisfied.[16] Consequently, $\delta \geq \frac{1}{2}$ is sufficient to ensure $\varphi \geq \frac{1}{2}$.

The first term in (39) is positive, because as shown above, $\frac{\partial \psi \psi}{\partial \eta} \leq 0$. For the second term to be also positive I need to specify the conditions which ensure $\frac{\partial S^{hh}}{\partial \eta} \geq 0$. This derivative is given by

$$\frac{\partial S^{hh}}{\partial \eta} = \frac{1}{\lambda_2^2 \lambda_3^2} \times \begin{bmatrix} (y\alpha_{hh} - y\alpha_{ll})[\gamma m(\theta)\lambda_2\lambda_3 + \gamma^3\eta^2 m(\theta)^3 + \gamma(1-\gamma)\eta(1-\eta)m(\theta)^3] \\ +(y\alpha_{hh} - b)\gamma(1-\gamma)(1-\eta)m(\theta)^2\lambda_2 + (\gamma y\alpha_{hh} - y\alpha_{ll})\gamma\eta m(\theta)^2\lambda_3 \end{bmatrix} \tag{40}$$

By rearranging terms in (40) one can verify that a sufficient, but not necessary condition for this derivative to be positive is $\frac{(y\alpha_{ll}-b)}{(y\alpha_{hh}-b)} \leq \gamma$. Therefore, when $\delta \geq \frac{1}{2}$ and $\frac{(y\alpha_{ll}-b)}{(y\alpha_{hh}-b)} \leq \gamma$, then $\frac{\partial F_h}{\partial \eta} \leq 0$.

[16]Note that when $s_h < s_l$, as it is well established in the data, this argument is reinforced.

References

Albrecht, J. and Vroman S. (2002), "A Matching Model with Endogenous Skill Requirements," *International Economic Review*, 43, 283-305.

Autor, L., Katz, F. and Kearney, M. (2008), "Trends in U.S. Wage Inequality: Revising the Revisionists," forthcoming in *Review of Economics and Statistics.*

Barlevy, G. (2002), "The Sullying Effect of Recessions," *The Review of Economic Studies*, 69, 65-96.

Beach, C. and Kaliski, S. (1987), " The Distribution of Unemployment Spells: Canada, 1978-82," *Industrial and Labor Relations Review,* 40(2), 254-267.

Belzil, C. (1996), "Relative Efficiencies and Comparative Advantages in Job Search," *Journal of Labor Economics*, 14(1), 154-173

Blanchard, D. and Diamond, P. (1990), "The Cyclical Behavior of the Gross Flows of U.S. Workers," *Brookings Papers on Economic Activity*, 1990(2), 85-155.

Blau, D. and Robins, P. (1990), "Job Search Outcomes for the Employed and the Unemployed," *Journal of Political Economy,* 98(3), 637-655.

Bowlus, A. (1995), "Matching Workers and Jobs: Cyclical Fluctuations in Match Quality," *Journal of Labor Econommics*, 13(2), 335-350.

Clark, K. and Summers, L. (1979), "Labor Market Dynamics and Unemployment: A Reconsideration," *Brookings Papers on Economic Activity*, 1979(1), 13-72.

Davis, S., Haltiwanger, J. and Schuh, S. (1996), *Job Creation and Destruction* (The MIT Press, Cambridge, MA).

Dolado, J., Jansen, M. and Jimeno, J. (2004),"On-the-job Search in a Matching Model with Heterogeneous Jobs and Workers" (mimeo, Universidad Carlos III de Madrid).

Fallick, B. and Fleischman, C. (2004). "Employer-to-Employer Flows in the U.S. Labor Market: the Complete Picture of Gross Worker Flows," *Finance and Economics Discussion Series, 2004-34*, (Board of Governors of the Federal Reserve System).

Gautier, P. (2002), "Unemployment and Search externalities in a model with Heterogeneous Jobs and Heterogeneous Workers," *Economica*, 69, 21-40.

Green, F., McInstosh, S. and Vignoles, A. (1999), "Overeducation and Skills-Clarifying the Concepts" (CEP discussion paper 435).

Hagedorn, M. and Manovskii, I. (2007), "The Cyclical Behavior of Unemployment and Vacnacies Revisited" (mimeo, University of Pennsylvania).

Hall, R. (2005),"Job Loss, Job Finding, and Unemployment in the U.S. Economy Over the Past Fifty Years" (National Bureau of Economic Research, Working Paper 11678).

Hamermesh, D. (1993) *Labor Demand* (Princeton Academic Press, Princeton, NJ).

Hecker, D. (1992), "Reconciling Conflicting Data on Jobs for College Graduates," *Monthly Labor Review*, July.

Hecker, D. (1995), "College Graduates in "High School" Jobs," *Monthly Labor Review*, December. Juhn, C., Murphy, K. and Topel, R. (2002), "Current Unemployment, Historically Contemplated," *Brookings Papers on Economic Activity,* 1, 79-116.

Krause, M. and Lubik, T. (2006), "The Cyclical Upgrading of Labor and On-the-job Search," *Labor Economics*, 13, 459-477.

Krause, M. and Lubik, T. (2007), "On-the-Job Search and Cyclical Dynamics of the Labor Market" (ECB Working Paper No. 779).

Manacorda, M. and Petrongolo, B. (1999), "Skill Mismatch and Unemployment in OECD Countries," *Economica,* 66(262), 181-207.

Mortensen, D. and Pissarides, C. (1994), "Job Creation and Job Destruction in the Theory of Unemployment," *The Review of Economic Studies*, 61(3), 397-415.

Nagypal, E. (2004), " Worker Reallocation over the Business Cycle: The Importance of Job-to-Job Transitions" (mimeo, Northwestern University).

Nickell, S. (1879), "Education and Lifitime Patterns of Unemployment," *Journal of Political Economy*, 87(5), 117-131.

Oliver, J. and Raymond, J. (2003), "Educacin Formal y Demanda de Cualificacin de laMano de Obra en Espaa: Una Visin A Largo Plazo," *Document d Economia Industrial* (Centre d Economia Industrial, Universitat Autnoma de Barcelona).

van Ours, J. and Ridder, G. (1995), "Job Matching and Job Competition: are Lower Educated Workers at the Back of Job Queues?," *European Economic Review*, 39, 1717-1731.

Petrongolo, B. (2001),"Reemployment Probabilities and Returns to Matching," *Journal of Labor Economics*, 19(3),716-741.

Petrongolo, B. and Pissarides, C. (2001), "Looking Into the Black Box: a Survey on the Matching Function," *Journal of Economic Literature*, 39(2),390-431.

Pissarides, C. and Wadsworth, J. (1994), "On-the-job Search: Some Empirical Evidence from Britain," *European Economic Review,* 38(2), 385-401.

Polsky, D. (1999), "Changing Consequences of Job Separation in the United States," *Industrial and Labor Relations Review*, 52(4), 564-580.

Pries, M. (2007), "Heterogeneity and Labor Market Volatility in Matching Models," forthcoming in *Review of Economic Dynamics.*

Royalty, A. (1998), "Job-to-job and Job-to-nonemployment Turnover by Gender and Education Level," *Journal of Labor Economics*, 16(2), 392-443.

Shimer, R. (2005a), "The Cyclicality of Hires, Separations, and Job-to-job Transitions," *Federal Reserve Bank of St. Louis Review*, 87(4),493-507.

Shimer, R. (2005b), "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review,* 95(1), 25-49.

Sicherman, N. (1991), "Overeducation in the Labor Market," *Journal of Labor Economics*, 9, 101-112.

Topel, R. (1984), "Equilibrium Earnings, Turnover, and Unemployment: New Evidence," *Journal of Labor Economics*, 2(4), 500-522.

Topel, R. (1993), “What have we Learned from Empirical Studies of Unemployment and Turnover?,” *The American Economic Review*, 83, 110-115.

Vishwanath, T. (1989), “Job Search, Stigma Effect, and Escape Rate from Unemployment,” *Journal of Labor Economics*, 7(4), 487-502.

Wheeler, C. (2005), “Evidence on Wage Inequality, Worker Education, and Technology,” *Federal Reserve Bank of St. Louis Review*, 87(3), 375-93.